

WIND SHEAR, ROUGHNESS CLASSES AND TURBINE ENERGY PRODUCTION

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INTRODUCTION

At a height of about 1 kilometer the wind is barely affected by the surface of the Earth. In the lower atmospheric layers the wind speed is affected by the friction against the surface of the Earth. One has to account for the roughness of the terrain, the influence of obstacles and the effect of the terrain contours, or the “Orography” of the area.

The more significant the roughness of the ground surface, the more the wind will be slowed down. Tall trees in woods and forests and large buildings in cities slow the wind considerably. Concrete runways at airports and road surfaces will marginally slow the wind. Water surfaces are smoother than concrete runways, and will have a lesser effect on the wind. Tall grass and shrubs and bushes will significantly slow down the wind.

The area around a potential wind turbine site rarely consists of a uniform homogeneous plain. Different crops, woods, forests, fence rows and buildings are usually scattered over the area. The end result is that the wind flow approaching the wind turbine sees a change in the surface roughness. The landscape up to a distance of 10 kilometers will have an impact at the turbine site, although the farther away the changes in roughness, the least impact they will cause.

With a knowledge of the average wind speed from measurements at a given location, the Rayleigh probability density function (pdf), or the Weibull pdf can be used in conjunction with a wind turbine rated power production curve to determine the energy production potential of a given wind turbine design at such a location. This becomes an important input to further economical assessments of the choice of alternate wind turbine designs for implementation at different wind production sites. This is considered as a better assessment methodology than just assigning an average intermittence or capacity factor for estimating the energy production potential of a wind turbine.

SURFACE ROUGHNESS AROUND WIND TURBINES

A change from a smooth to a rough site will increase the surface frictional stress and consequently lead to the wind at the surface slowing down. This increased shear will thus lead to the effect being slowly passed upwards through the surface boundary layer until the wind throughout the layer is slowed down. The boundary layer gets back into equilibrium with the surface.

A rough to a smooth surface change will lead to a speeding up of the wind throughout the profile. To reach equilibrium there must be a sufficiently long stretch of ground.

Other roughness changes will occur before equilibrium can be reached. As a result, an internal boundary layer is formed which grows with the distance downstream. At any distance downstream where equilibrium was not been reached there will be a kink in the

wind profile at some critical height h_{crit} . At a lower height than h_{crit} , the profile would be disturbed by the change in the surface. Above h_{crit} the profile would be still determined by the initial surface roughness.

If the driving force of the wind or the geostrophic wind is the same over the area under consideration, it is possible to still use a modified logarithmic law to describe the profile. Each subsequent roughness change can be treated in the same way, as long as the changes do not occur over too short a distance.

In addition to the changes in surface roughness, a change in the surrounding landscape can create a change in the surface temperature or moisture fluxes which can impact the shape of the profile. This is apparent for changes from sea to land or ice to land. The effect of this is to alter the stability of the wind profile, and the vertical movement of the air becomes more important.

ROUGHNESS CLASSES AND LENGTHS

Roughness classes and roughness lengths are characteristics of the landscape used to evaluate wind conditions at a potential wind turbine site. A high roughness class of 3-4 characterizes landscapes dotted with trees and buildings. A sea surface has a roughness class of zero.

Concrete runways at airports and road surfaces have a roughness class of 0.5. This is also the case of a flat and open landscape grazed by cattle and livestock. Siting wind mills on grazed land appears to be advantageous since grazing reduces the surface roughness of the surrounding ground.

The roughness length is defined as the height above ground Z_0 in meters at which the wind speed is theoretically equal to zero.

The Roughness Classes (RCs) are defined in terms of the roughness length in meters Z_0 , according to:

$$\begin{aligned}
 RC &= 1.699823015 + \frac{\ln Z_0}{\ln 150}, & \text{for } Z_0 \leq 0.03 \\
 RC &= 3.912489289 + \frac{\ln Z_0}{\ln 3.3333}, & \text{for } Z_0 > 0.03
 \end{aligned}
 \tag{1}$$

Table 1. Roughness classes and the associated roughness lengths.

Roughness Class RC	Roughness Length, Z_0 [m]	Energy Index [percent]	Landscape
0	0.0002	100	Water surface.
0.5	0.0024	73	Completely open terrain with a smooth surface, such as concrete runways in airports, mowed grass.

1	0.03	52	Open agricultural area without fences and hedgerows and very scattered buildings. Only softly rounded hills
1.5	0.055	45	Agricultural land with some houses and 8 meter tall sheltering hedgerows within a distance of about 1,250 meters.
2	0.1	39	Agricultural land with some houses and 8 meter tall sheltering hedgerows within a distance of about 500 meters.
2.5	0.2	31	Agricultural land with many houses, shrubs and plants, or 8 meters tall sheltering hedgerows within a distance of about 250 meters.
3	0.4	24	Villages, small towns, agricultural land with many or tall sheltering hedgerows, forests and very rough and uneven terrain.
3.5	0.8	18	Larger cities with tall buildings.
4	1.6	13	Very large cities with tall buildings and sky scrapers.

WIND SHEAR

The wind speed profile trends to a lower speed as we move closer to the ground level. This is designated as wind shear.

The wind speed at a certain height above ground can be estimated as a function of height above ground z and the roughness length Z_0 from Table 1 in the current wind direction from the formula:

$$V(z) = V_{ref} \frac{\ln \frac{z}{Z_0}}{\ln \frac{z_{ref}}{Z_0}} \quad (2)$$

The reference speed V_{ref} is a known wind speed at a reference height z_{ref} .

The formula assumes neutral atmospheric stability conditions under which the ground surface is neither heated nor cooled compared with the air temperature.

EXAMPLE

Considering a wind blowing at $V_{ref} = 8$ m/sec at a height of $z_{ref} = 20$ m. We can calculate the wind speed at the hub height of a wind turbine at a height of 50 m in a roughness class 2 agricultural land with some houses and 8 meter tall sheltering hedgerows within a distance of about 500 meters with a roughness length of $Z_0 = 0.1$ from:

$$V(50) = 8 \frac{\ln \frac{50}{0.1}}{\ln \frac{20}{0.1}} = 8 \frac{6.214}{5.298} = 9.38 \text{ [m/sec]}$$

The velocity profile as a function of height is shown in Fig. 1.

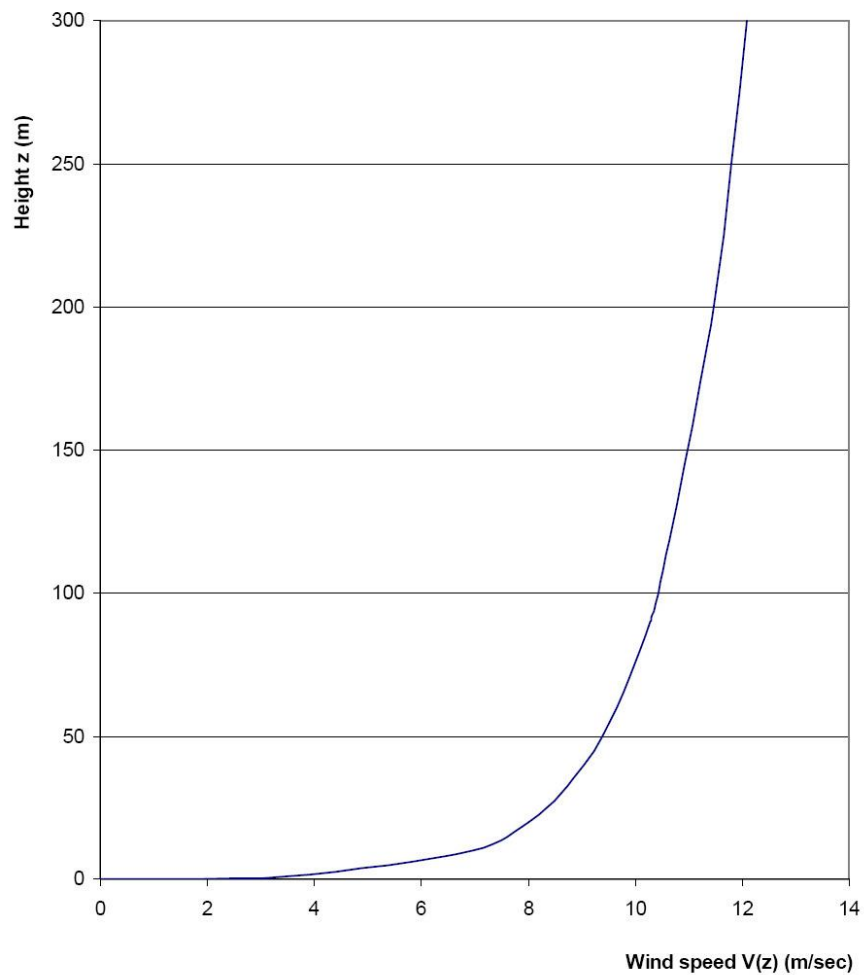


Figure 1. Velocity profile as a function of height for a roughness class 2 agricultural land with some houses and 8 meter tall sheltering hedgerows within a distance of about 500 meters with a roughness length of $Z_0 = 0.1$.

DESIGN IMPLICATIONS

Wind shear becomes important when designing wind turbines. If we consider a wind turbine with a hub height of 50 meters and a rotor diameter of 40 meters, we can calculate that the wind is blowing at a speed of:

$$V(70) = 8 \frac{\ln \frac{70}{0.1}}{\ln \frac{20}{0.1}} = 8 \frac{6.551}{5.298} = 9.89 \text{ [m/sec]}$$

when the tip of the blade is in its uppermost position $z = 50 + 20 = 70$ m, and a lower value of:

$$V(30) = 8 \frac{\ln \frac{30}{0.1}}{\ln \frac{20}{0.1}} = 8 \frac{5.703}{5.298} = 8.61 \text{ [m/sec]}$$

when the tip is in the bottom position at $z = 50 - 20 = 30$ m height.

This suggests that the forces acting on the rotor blade when it is in its top position are larger than when it is in its bottom position.

THE ROUGHNESS ROSE

If the wind speed were accurately measured at hub height over an extended period at the spot where a wind turbine will be standing, one can make an exact prediction of energy production. Since this is not possible, practically we have to depend on wind measurements made elsewhere in the area such as at an airport site. This can be depended on except in cases with a very complex terrain that is hilly or dotted with obstacles.

In the same way that a wind rose is used to map the amount of wind energy coming from different directions, a roughness rose is used to describe the roughness of the terrain in different directions to a prospective wind turbine site.

For a roughness rose, usually the compass is divided into 12 sectors of 30 degrees each that match the used wind rose.

For each sector an estimate of the roughness of the terrain is made, using the definitions from Table 1. The average wind speed is affected by the different roughness classes of the terrain.

If the roughness of the terrain does not exactly fall into any of the roughness classes, an averaging process would have to be adopted in the prevailing wind directions.

LANDSCAPE WITHOUT NEUTRAL STABILITY

The previous analysis assumed neutral stability of the atmosphere meaning that a parcel of air is adiabatically balanced from a thermodynamic perspective. If a parcel of air were displaced up or down, it would expand or contract without losing or gaining internal energy and still be in balance with the surrounding atmosphere.

Neutral stability is a reasonable assumption in high wind when shearing forces rather than buoyancy forces are dominant. However the atmosphere is rarely neutral and the buoyancy forces usually predominate over the shear forces as noticed from the scattering of the measurements at neighboring data collection points.

The atmosphere may be in a stable or an unstable condition. Stable boundary layers are encountered at night when the ground is cooled and the air tends to sink down. At night time, the boundary layers tend to be shallow at about 30 m in thickness and turbulence is suppressed. Under this condition wind speeds are low.

The unstable boundary layers are usually the deepest reaching up to 2 km in height on an active convective summer day. The ground is heated by the sun and the air rises. There are fluxes of moisture which depend on the humidity of the atmosphere and the moisture available at the surface. These factors combine to affect the wind profile in the lowest few tens of meters in the atmosphere.

The neglect of the stability effects particularly at coastal or offshore sites could significantly alter the assessment of a wind resource. The additional large scale turbulence under unstable conditions is also important in the siting and operational considerations.

MODIFIED LOG LAW

The logarithmic wind profile equation in the lowest 100 meter may still be used under non-equilibrium conditions with some appropriate modifications:

$$V(z) = \frac{V_{fr}}{k} \left[\ln\left(\frac{z}{Z_0}\right) - \psi\left(\frac{z}{L}\right) \right] \quad (3)$$

where: V_{fr} is the friction velocity,

$k = 0.4$ is the von Karman constant,

L is the Monin-Obukhov length,

Z_0 is the roughness length.

The Monin-Obukhov length L is a scaling parameter which depends upon the heat flux at the ground surface q_0 , and is given by:

$$L = \frac{T_0}{k \cdot g} \frac{c_p V_{fr}^3}{q_0} \quad (4)$$

where: T_0 is the ground surface absolute temperature, $^{\circ}\text{K} = 273 + ^{\circ}\text{C}$,

q_0 is the ground surface heat flux,

c_p is the heat capacity of the air at constant pressure,

g is the gravity's acceleration.

Different forms of the correction term have been suggested for stable and unstable conditions. Under unstable conditions it is given by:

$$\psi_{unstable}\left(\frac{z}{L}\right) = \left(1 - 16\frac{z}{L}\right)^{1/4} - 1 \quad (5)$$

Under stable conditions, the correction term is given by:

$$\psi_{stable}\left(\frac{z}{L}\right) = -4.7\frac{z}{L} \quad (6)$$

Under neutral conditions, the Monin-Obukhov length L is infinite in magnitude and the correction term vanishes.

WIND POWER FLUX CLASSIFICATION

The wind resources at different locations are rated according to its available power flux or power available per square meters of rotor area. The power flux units are in Watts/m², but it should be noted that it is sometimes referred to as power density, which would have units of Watts/m³.

A classification by the Battelle Pacific Northwest Laboratories is shown in Table 2. A good wind resource location would be a class 7 designation. A minimum of a class 3 designation is needed for considering a site as economically viable.

The power flux is usually reported at a reference height of 10 meters which is related to meteorological measurements for instance at airport sites, and is then extrapolated to a height of 50 meters, which would be the hub height of an industrial wind turbine.

The extrapolation for the power flux from the 10 meters meteorological height to the 50 meters wind turbine hub height is usually achieved using a so-called 1/7 power law for undisturbed air given by:

$$\frac{P_1}{P_2} = \left(\frac{Z_1}{Z_2}\right)^{3.7} \quad (7)$$

The mean wind speed is based on the consideration of a Rayleigh speed probability density function at standard sea level. To adjust for the same power flux the speed increases 3 percent per 1,000 m of elevation, or 5 percent per 5,000 ft of elevation.

Table 2. Wind power flux classes at 10 and 50 meters height.

Wind Power Flux Class	At 10 meters (33 ft) height			At 50 meters (164 ft) height		
	Power flux [Watt/m ²]	Wind speed [m/s]	Wind speed [mph]	Power flux Watt/m ²	Wind speed [m/s]	Wind speed [mph]
1	0-100	0-4.4	0-9.8	0-200	0-5.6	0-12.5
2	100-150	4.4-5.1	9.8-11.5	200-300	5.6-6.4	12.5-14.3

3	150-200	5.1-5.6	11.5-12.5	300-400	6.4-7.0	14.3-15.7
4	200-250	5.6-6.0	12.5-13.4	400-500	7.0-7.5	15.7-16.8
5	250-300	6.0-6.4	13.4-14.3	500-600	7.5-8.0	16.8-17.9
6	300-400	6.4-7.0	14.3-15.7	600-800	8.0-8.8	17.9-19.7
7	400-1000	7.0-9.4	15.7-21.1	800-2000	8.8-11.9	19.7-26.6

JUSTIFICATION FOR THE NECESSITY TO USE A WIND DURATION CURVE

Consider a turbine at a hub height of 50 meters from Table 2 and a rotor swept area of 100 m².

Over a 24 hours period consider that the wind blows at 8.8 m/s for 12 hours and does not blow at all for the other 12 hours.

The energy produced over the 24 hours period will be according to Table 2:

$$\begin{aligned}
 E_1 &= (800 \frac{\text{watts}}{\text{m}^2} \times 100\text{m}^2 \times 12\text{hrs}) + (0 \frac{\text{watts}}{\text{m}^2} \times 100\text{m}^2 \times 12\text{hrs}) \\
 &= 960,000[\text{watts.hr}] \\
 &= 960[\text{kW.hr}]
 \end{aligned}$$

Using the average value of wind speed over the 24 hours period as:

$$\frac{(8.8 \times 12) + (0 \times 12)}{24} = 4.4[\text{m/s}]$$

would yield the incorrect value of energy production over a 24 hours period according to Table 2 as:

$$\begin{aligned}
 E_2 &= 200 \frac{\text{watts}}{\text{m}^2} \times 100\text{m}^2 \times 24\text{hrs} \\
 &= 480,000[\text{watts.hr}] \\
 &= 480[\text{kW.hr}]
 \end{aligned}$$

This suggests the need to take into account the distribution of wind speeds to correctly estimate the energy potential of a wind turbine at a given location, not just the average wind speed at the location.

RAYLEIGH PROBABILITY DENSITY FUNCTION

As an alternative to the Weibull probability density function, the Rayleigh probability density function is used to model a two dimensional vector such as wind velocity consisting of a speed magnitude and a direction value that are normally distributed, uncorrelated and have an equal variance. The magnitude of the velocity vector or speed

can then be described by a Rayleigh distribution:

$$R(v; \sigma)dv = \frac{v}{\sigma^2} e^{-\frac{v^2}{2\sigma^2}} dv$$

where: dv is the speed granularity, (8)
 $dv=1$ for speed intervals of 1,
 $dv=2$ for speed intervals of 2.

which has a mean speed value of:

$$\bar{v} = \sqrt{\frac{\pi}{2}} \sigma \quad (9)$$

and a variance:

$$\text{var}(v) = \frac{4 - \pi}{2} \sigma^2 \quad (10)$$

From Eqn. 9, we can write:

$$\sigma^2 = \frac{2}{\pi} \bar{v}^2 \quad (11)$$

Substituting in Eqn. 8, we obtain an alternate form of the Rayleigh probability density function as:

$$R(v ; \bar{v}) = \frac{\pi}{2} \left(\frac{v}{\bar{v}^2} \right) e^{-\frac{\pi}{4} \left(\frac{v}{\bar{v}} \right)^2} \quad (12)$$

WIND ENERGY PRODUCTION POTENTIAL AT A GIVEN SITE

Different sites possess different energy potential capabilities. For instance the maximum and average wind speeds in miles per hour at the Willard airport at Champaign, Illinois, measured at 33 ft of height over the period of 1989-2005 is shown in Fig. 2.

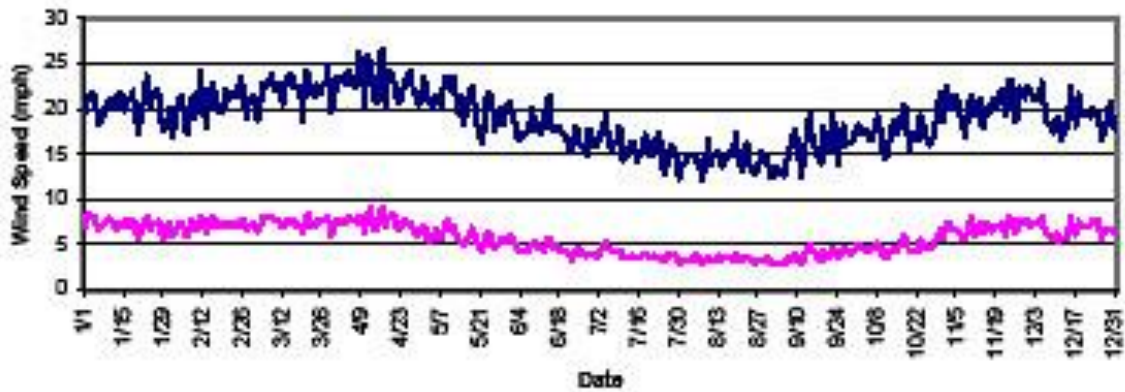


Figure 2. Maximum and average wind speed measurements at the Willard airport at Champaign, Illinois measured at a height of 33 feet over the period 1989-2005.

For a class 7 power flux location at 50 meters height and with an average wind speed of about 22.3 mph or 10 m/s, the Rayleigh probability density function takes the form:

$$R(v) = \frac{\pi}{2} \left(\frac{v}{100} \right) e^{-\frac{\pi}{4} \left(\frac{v}{10} \right)^2}$$

$$= 0.01570796ve^{-0.00785398v^2}$$

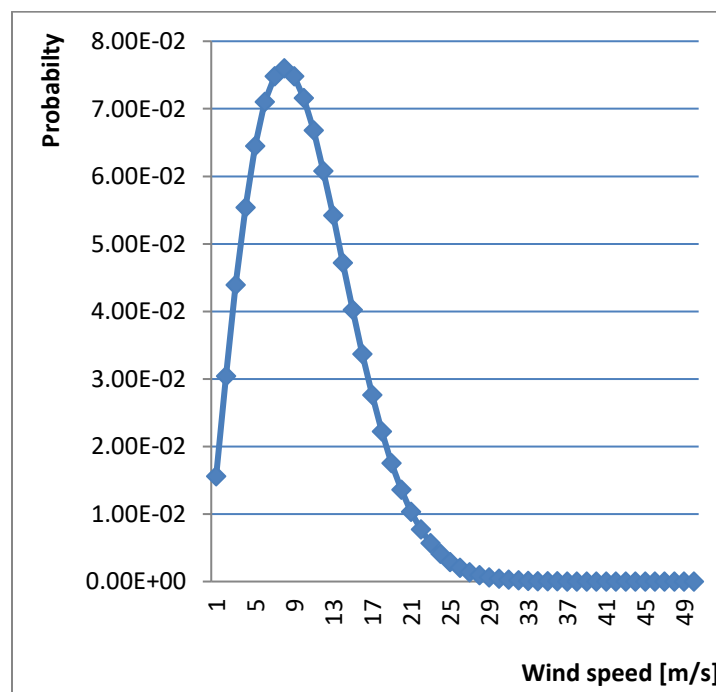


Figure 3. Rayleigh probability density function for an average wind speed of 10 m/s.

This has to be convoluted into the rated power curve provided by the manufacturer of a given turbine design, to estimate the energy production potential of a turbine design at a given site:

$$E(v) = \int_0^t R(v,t)P(v,t)dt$$

$$\approx \sum_i R_i(v)P_i(v)\Delta t_i \left[\frac{kW \cdot hr}{yr} \right]$$

where: $R_i(v)$ is the probability of duration at speed v from the Rayleigh pdf (13)

$R_i(v)\Delta t_i = R_i(v) \times 8760$ is the yearly duration at speed v $\left[\frac{hrs}{yr} \right]$

$P_i(v)$ is the turbine power production at speed v from its power curve [kW]

where the integration was approximated by a summation.

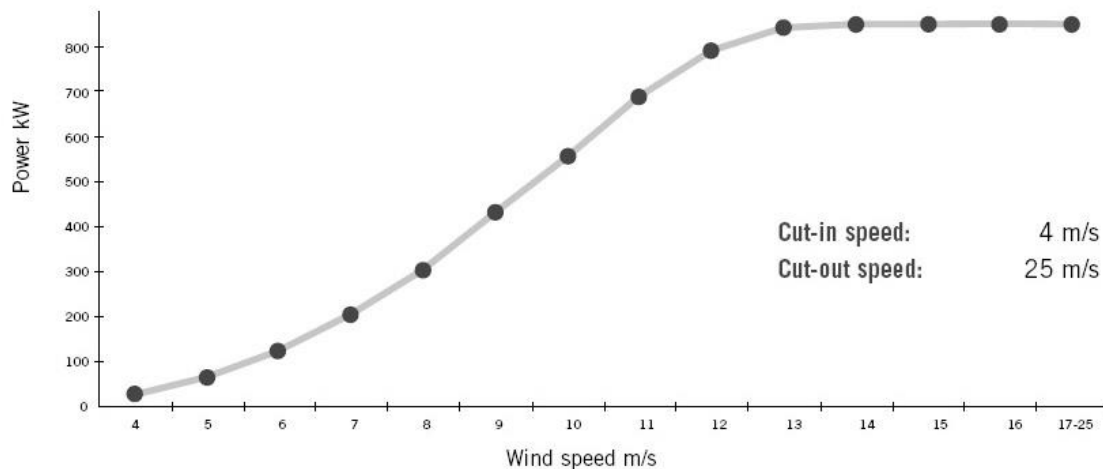


Figure 4. Power curve of the Gamesa G52 850 kW wind turbine. Source: Gamesa.

Table 3. Power curve data for the Gamesa G52-850 kW wind turbine.

Power $P(v)$ [kW]	Wind speed v [m/s]
27.9	4
65.2	5
123.1	6
203.0	7
307.0	8
435.3	9

564.5	10
684.6	11
779.9	12
840.6	13
848.0	14
849.0	15
850.0	16
850.0	17-25

Combining the wind measurements data incorporated into the Raleigh probability density function into the rated power curve, one can thus estimate the energy production potential of a turbine design at a given site as an input for an economical assessment of the project under consideration.

Table 4. Energy production potential from a typical wind turbine at a potential site with an average wind speed of 10 m/s.

Wind speed v [m/s]	Rayleigh pdf probability $R_i(v)$	Duration time per year at speed v^* $R_i(v) \times 8760$ [hr/yr]	Rated power at speed $P_i(v)$ [kW]	Maximum possible yearly energy production $E(v)$ [kW.hr/yr]
1	1.56E-02	1.37E+02		
2	3.04E-02	2.67E+02		
3	4.39E-02	3.85E+02		
4 (cut-in)	5.54E-02	4.85E+02	27.9	13,532
5	6.45E-02	5.65E+02	65.2	36,838
6	7.10E-02	6.22E+02	123.1	76,568
7	7.48E-02	6.56E+02	203.0	133,168
8	7.60E-02	6.66E+02	307.0	204,462
9	7.48E-02	6.56E+02	435.3	285,557
10	7.16E-02	6.27E+02	564.5	353,942
11	6.68E-02	5.85E+02	684.6	400,491
12	6.08E-02	5.33E+02	779.9	415,687
13	5.42E-02	4.74E+02	840.6	398,444
14	4.72E-02	4.13E+02	848.0	350,224
15	4.02E-02	3.53E+02	849.0	299,697
16	3.37E-02	2.95E+02	850.0	250,750
17	2.76E-02	2.42E+02	850.0	205,700
18	2.22E-02	1.94E+02	850.0	164,900
19	1.75E-02	1.53E+02	850.0	130,050
20	1.36E-02	1.19E+02	850.0	101,150
21	1.03E-02	9.05E+01	850.0	76,925
22	7.72E-03	6.76E+01	850.0	57,460

23	5.67E-03	4.97E+01	850.0	42,245
24	4.09E-03	3.58E+01	850.0	30,430
25 (cut-off)	2.90E-03	2.54E+01	850.0	21,590
26	2.02E-03	1.77E+01		
27	1.38E-03	1.21E+01		
28	9.31E-04	8.16E+00		
29	6.16E-04	5.40E+00		
30	4.01E-04	3.51E+00		
31	2.57E-04	2.25E+00		
32	1.62E-04	1.42E+00		
33	1.00E-04	8.76E-01		
34	6.09E-05	5.33E-01		
35	3.65E-05	3.19E-01		
36	2.15E-05	1.88E-01		
37	1.24E-05	1.09E-01		
38	7.09E-06	6.21E-02		
39	3.97E-06	3.48E-02		
40	2.19E-06	1.92E-02		
41	1.19E-06	1.04E-02		
42	6.35E-07	5.56E-03		
43	3.33E-07	2.92E-03		
44	1.72E-07	1.51E-03		
45	8.75E-08	7.67E-04		
46	4.38E-08	3.84E-04		
47	2.16E-08	1.89E-04		
48	1.04E-08	9.14E-05		
49	4.97E-09	4.36E-05		
50	2.33E-09	2.04E-05		
Total	0.998690	8748.525		4,049,810

* 1 year = 365 x 24 = 8,760 hrs

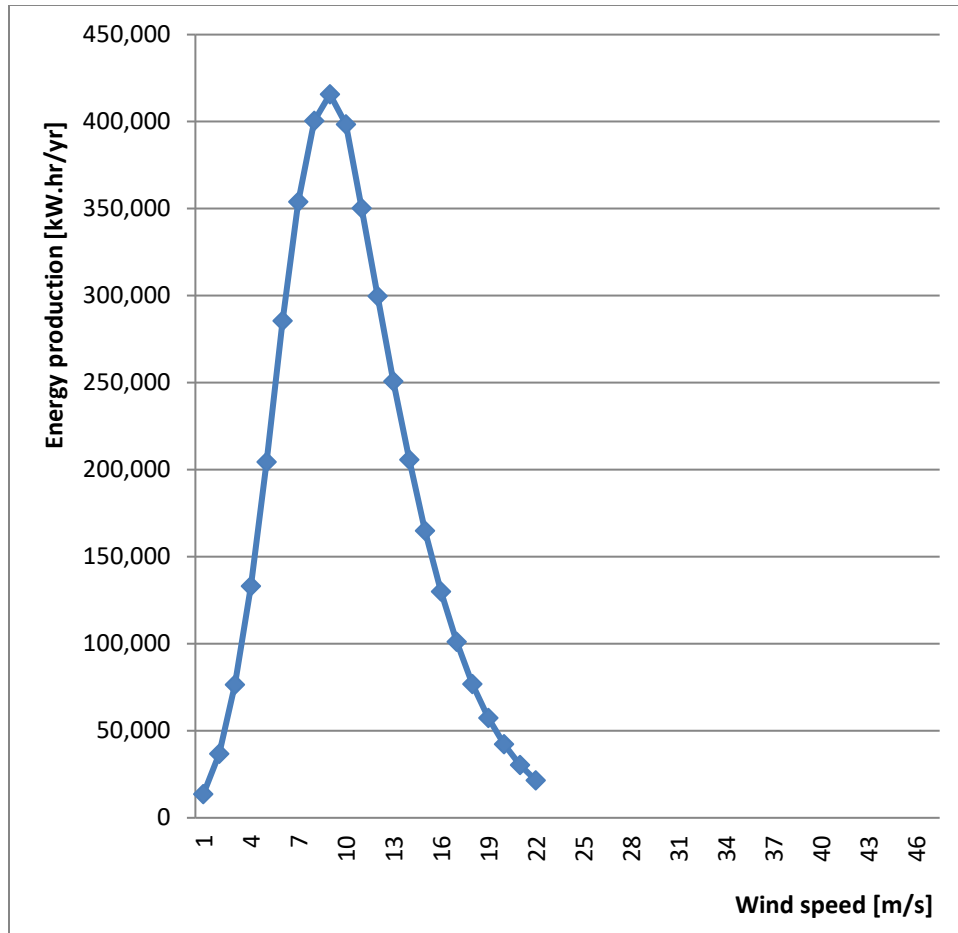


Figure 5. Maximum possible yearly energy production as a function of wind speed.

The potential energy production is now the integral or summation over all speeds, with the speed intervals taken as unity for simplification:

$$\begin{aligned}
 E &= \int_0^{\infty} E(v)dv \\
 &\approx \sum_i E_i(v)\Delta v_i
 \end{aligned}
 \tag{14}$$

It can be noticed that the turbine energy production curve follows the Rayleigh distribution of wind speeds more closely than the rated power production curve of the turbine. In fact, the turbine is operating primarily in the low wind speed region.

DISCUSSION

The energy production potential is estimated assuming no turbulence caused by the presence of trees or buildings. A rule of thumb to avoid the turbulence effects is the placement of the hub of a wind turbine at least 30 feet above an obstruction within a radius

of 200 feet from the base of its structural tower.

The estimated energy production is a maximum potential value and is lowered in practice by the limitations of the rotors design as well as the efficiencies of the transmission and electrical generator systems.

It is important to measure the wind speed variation for a given location for at least a year, due to the seasonal variations, for an adequate assessment of its economical wind production potential.

EXERCISES

1. Plot the wind shear profile as a function of height for a wind blowing at $V_{\text{ref}} = 8$ m/sec at a height of $z_{\text{ref}} = 20$ m on water and on agricultural land with some houses and 8 meter tall sheltering fence rows within a distance of about 1,250 meters.
2. A Japan Steel Works (JSW) J82-2.0 / III wind turbine has a rotor blade length of 40 m. Estimate the wind speed at the tips of its blades at the maximum and minimum heights they attain, if the hub height is:
 - a. 65 meters.
 - b. 80 meters.

Assume the turbine is built within an area with a roughness class of 2.5, for a wind blowing at $V_{\text{ref}} = 8$ m/sec at a height of $z_{\text{ref}} = 20$ m.

3. a) Estimate the average wind speed at the Champaign Willard airport location.
 - b) Determine its wind class classification.
 - c) Plot the corresponding Rayleigh probability density function.
 - d) Using the power curve for the Gamesa G52-850 kW wind turbine, generate the graph of the potential energy production as a function of wind speed.
 - e) Estimate the yearly total energy production.
 - f) Compare the total potential energy production for this wind class site to that obtainable from a wind class 7 location.
4. Prove that the cumulative distribution function (cdf) of the Rayleigh pdf is given by:

$$cdf : \left[1 - e^{\left(-\frac{v^2}{2\sigma^2} \right)} \right] dv$$

REFERENCES

1. _____, "Water and Atmospheric Resources Monitoring Program," Illinois Climate Network, Illinois State Water Survey, Champaign, Illinois, 2006.
2. Paul Gipe, "Wind Power," White River Junction, Vermont: Chelsea Green, 2004.