

# ENERGY AND POWER CONTENT OF THE WIND

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## INTRODUCTION

A unique feature of a wind turbine is that the energy extraction process uses a change in the wind speed, and not in the temperature like in the case of a heat engine such as a steam or a gas turbine, a change in the head of a hydraulic system, or the change in potential of an electrical system.

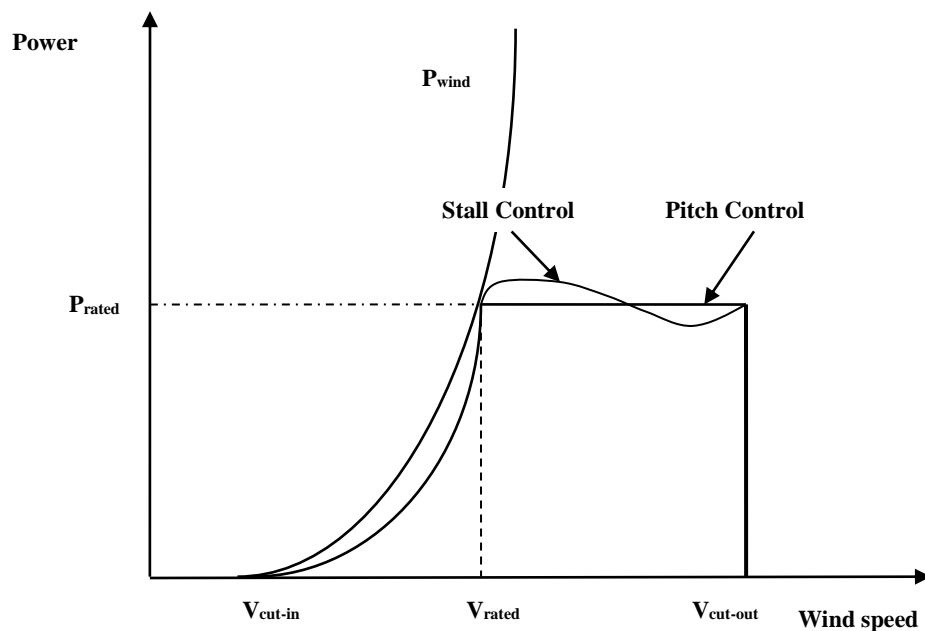


Figure 1. Power content in the wind and the extractable rated power for pitch controlled and stall controlled wind turbines, as a function of wind speed.

Similarly, only a fraction of the energy content in the wind is extractable. The power curve of a wind turbine shows that it starts rotation at a cut-in wind speed, reaches its rated power output at a rated design wind speed, and then shuts off at a cut-out wind speed. Operation beyond the cut-out wind speed could lead to a runaway turbine condition, resulting in its damage or even its disintegration.

Wind speed and direction are important design parameters in wind power systems. They both vary in the short and long terms. In the short term, wind gusts can occur over intervals of seconds. Average wind speeds vary according to the time of the day or the prevailing season. On the long term, some years could be windy, whilst others may be calmer than average, depending on the climatic conditions. Terrain, geographical features, tree lines and height above ground also affect the wind speed.

The power of a wind turbine is shown to vary proportional the cube of the wind speed over most of its operational range. Thus a doubling the wind speed increases the power by a factor of  $2^3 = 8$  times. Overestimating the wind speed could mean that less power is available than expected, negatively affecting the economics of a wind energy project. Underestimating the wind speed may be less significant, but could mean that a wind turbine with a larger generator should have been installed, missing on possible cost savings.

At times when the wind does not blow, wind turbines cannot rotate and will not produce power. When the wind blows, they will be able to deliver full output. Under unfavorable conditions such as severe storms, tornadoes or hurricanes, wind turbines must be shut down, or their power production reduced, lest they would be subject to a runaway condition resulting in catastrophic failure.

## **WIND SPEED FLUCTUATION, TURBULENCE INTENSITY FACTOR**

The instantaneous wind speed can be described as an average wind speed and an added fluctuating perturbation as:

$$V = \bar{V} + \delta \quad (1)$$

The mean wind speed is usually determined at 10 minutes interval values. The fluctuating component of the wind flow is expressed in terms of the root mean square (rms) value of the fluctuating term as the dimensionless Turbulence Intensity Factor (T) given by:

$$T = \frac{\sqrt{\delta^2}}{\bar{V}} = \frac{1}{\bar{V}} \sqrt{\frac{\int_0^{\tau} \delta^2 dt}{\tau}} \quad (2)$$

For rough terrain with structures and obstructions, T is in the range 0.15-0.20. For smooth terrain such as the water surface it is about 0.10.

## **VERTICAL WIND PROFILE**

At the surface, the wind speed is zero. It increases with height rapidly near the ground and increases less rapidly as it reaches higher altitudes. It reaches a plateau at a height of 2 km and does not increase any more. The change in wind speed as a function of height is determined experimentally and fits different mathematical functions.

The power exponent function can express the wind speed as a function of the height above ground level z as:

$$V(z) = V_r \left( \frac{z}{z_r} \right)^a \quad (3)$$

$z_r$  = reference height  
 $V_r$  = speed at  $z_r$   
 $a$  = terrain roughness exponent

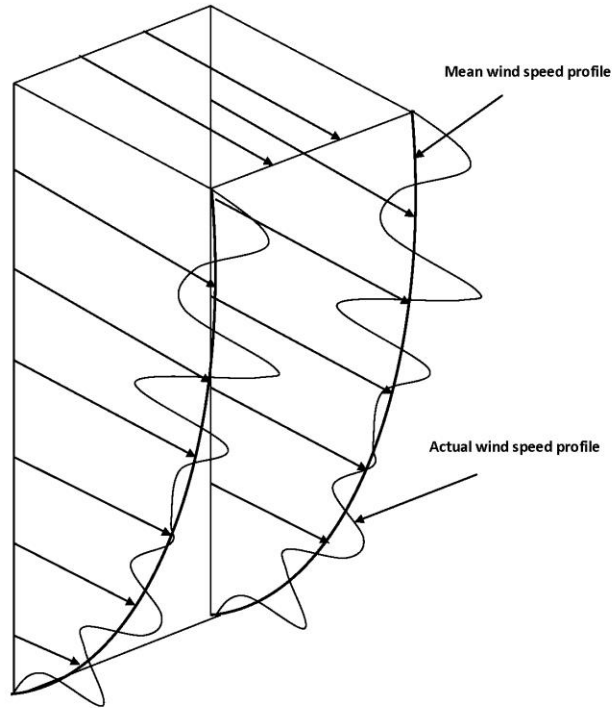


Figure2. Wind speed profile  $V(z)$  as a function of height  $z$ .

### EXAMPLE

If the wind speed at 10 meters of height is 20 m/s, one can deduce the wind speed at 40 m if  $a = 0.1$  as:

$$V(40) = 20 \left( \frac{40}{10} \right)^{0.1} = 20 \times 4^{0.1} = 23 \left[ \frac{\text{m}}{\text{s}} \right].$$

The logarithmic function form is also used to express the wind speed in the form:

$$V(z) = V(10) \frac{\ln\left(\frac{z}{z_0}\right)}{\ln\left(\frac{10}{z_0}\right)} \quad (4)$$

$z_0$  = roughness length

For different types of terrain, the values of the parameters  $\alpha$  and  $z_0$  are shown in Table 1.

Table 1. Vertical wind profile parameters.

Terrain type	Roughness class	Roughness length, $z_0$ [m]	Exponent $a$
Water surface	0	0.001	0.10
Open country	1	0.012	0.12
Farmland with structures and hedges	2	0.050	0.16
Farmland with tress, woods and villages	3	0.300	0.28

## KINETIC ENERGY OF THE UNDISTURBED WIND STREAM CONSERVATION OF ENERGY

The fundamental concept of wind kinetic energy  $E$  is first considered. The kinetic energy in an air flow of mass  $m$  through a unit area perpendicular to the wind direction, assuming a constant flow velocity  $V$  [m/s] is:

$$E = \frac{1}{2} m V^2 \text{ [Joules]} \quad (5)$$

where:  $V$  is the wind speed in [m/s],  
 $m$  is the wind mass in [kg].

The kinetic energy per unit volume  $v$  or energy density is thus:

$$\begin{aligned} E' &= \frac{1}{2} \frac{m}{v} V^2 \\ &= \frac{1}{2} \rho V^2 \left[ \frac{\text{Joules}}{\text{m}^3} \right] \end{aligned} \quad (6)$$

where:  $\rho = \frac{m}{v}$  is the air density in [kg/m<sup>3</sup>],  
 $v$  is the considered air volume in [m<sup>3</sup>].

The specific wind energy or kinetic energy per unit mass is given from Eqn. 1 as:

$$E'' = \frac{E}{m} = \frac{1}{2}V^2 \left[ \frac{\text{Joules}}{\text{kg}} \right] \quad (7)$$

The specific wind energy is proportional to the square of the wind speed.

## **WIND POWER DENSITY, ENERGY FLUX CONSERVATION OF MASS EQUATION**

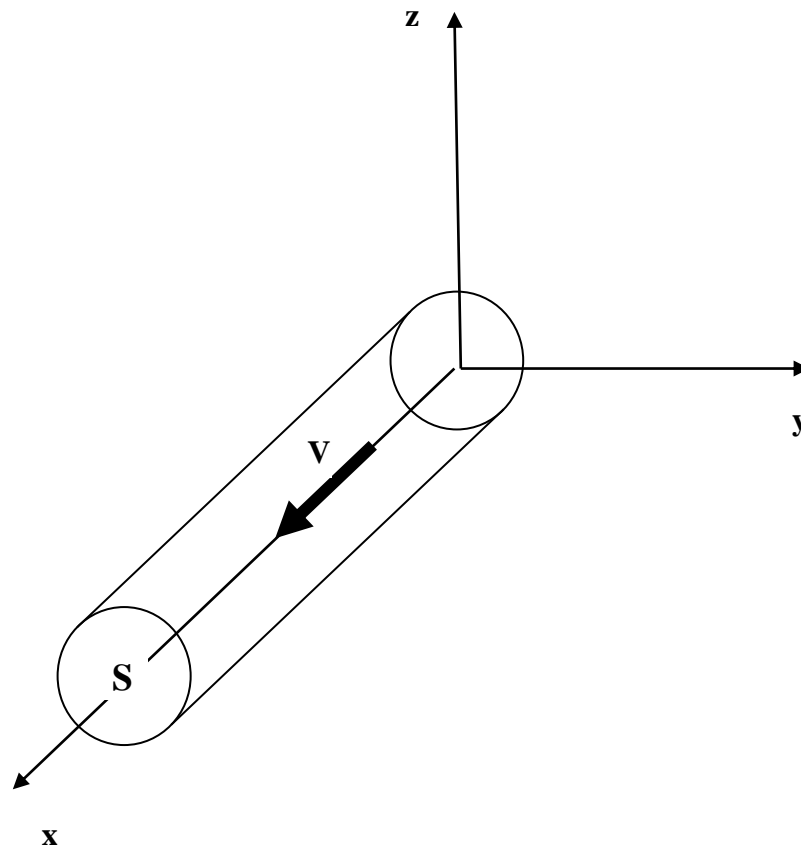


Figure 3. Undisturbed wind stream cylinder moving at a speed  $V$  in the  $x$  direction.

For an air stream flowing through a cross sectional area  $S$  [ $\text{m}^2$ ], the mass flow rate is:

$$\dot{m} = \rho SV, \left[ \frac{\text{kg}}{\text{m}^3} \cdot \text{m}^2 \cdot \frac{\text{m}}{\text{sec}} \right], \left[ \frac{\text{kg}}{\text{sec}} \right]. \quad (8)$$

Since the power content of the air stream is its energy content per unit time:

$$\begin{aligned}
P &= \frac{dE}{dt}, \left[ \frac{\text{Joules}}{\text{sec}} \right], [\text{Watts}], \\
&= \frac{d\left(\frac{1}{2}mV^2\right)}{dt} \\
&= \frac{dm}{dt} \cdot \frac{1}{2}V^2 \\
&= \dot{m} \cdot \frac{1}{2}V^2, \text{ for constant } V
\end{aligned} \tag{9}$$

Thus we can write that the power from Eqn. 7 is:

$$\begin{aligned}
P &= [\text{mass flow rate}] \cdot [\text{kinetic energy per unit mass}] \\
&= \dot{m} \cdot E', \left[ \frac{\text{kg}}{\text{sec}} \cdot \frac{\text{Joules}}{\text{kg}} \right], \left[ \frac{\text{Joules}}{\text{sec}} \right], [\text{Watts}]
\end{aligned} \tag{10}$$

From Eqns. 7 and 8, we can also write for the power P:

$$\begin{aligned}
P &= \rho SV \cdot \frac{1}{2}V^2, \left[ \frac{\text{kg}}{\text{sec}} \cdot \frac{\text{Joules}}{\text{kg}} \right] \\
&= \frac{1}{2} \rho SV^3, \left[ \frac{\text{Joules}}{\text{sec}} \right], [\text{Watts}]
\end{aligned} \tag{11}$$

The swept area S can be expressed in terms of its radius R as:

$$S = \pi R^2$$

This leads to an alternate form of Eqn. 11 as:

$$P = \frac{1}{2} \rho \pi R^2 V^3, [\text{Watts}] \tag{11}'$$

The power flux (sometimes erroneously referred to as power density) is defined as:

$$\begin{aligned}
P' &= \frac{P}{S} \\
&= \frac{\frac{1}{2} \rho S V^3}{S} \\
&= \frac{1}{2} \rho V^3, \left[ \frac{\text{Joules}}{\text{m}^2 \cdot \text{sec}} \right], \left[ \frac{\text{Watts}}{\text{m}^2} \right].
\end{aligned} \tag{12}$$

We notice that the power flux or power density is proportional to the cube of the wind speed. Higher wind speeds imply a higher power content of the wind stream.

## TOTAL AVAILABLE ENERGY

The energy passing through an area A [m<sup>2</sup>] during time t, from Eqn. 9, is given by:

$$\begin{aligned}
E &= \int_0^t P \cdot dt \\
&= \frac{1}{2} \rho S V^3 \cdot t, [\text{Watts} \cdot \text{sec}], [\text{Joules}].
\end{aligned} \tag{13}$$

This constitutes the total energy available for doing work on a wind turbine.

Like in water turbines, there is practically no change in the temperature of the water or air passing through the turbine. The energy extracted here is using a change in the working fluid velocity, and not its temperature like in the case of a heat engine such as a steam or a gas turbine.

## EFFECT OF PRESSURE, TEMPERATURE AND AIR DENSITY

The air density  $\rho$  as a function of its barometric pressure P [mm of mercury (Hg)] and temperature T [kelvin, K], is given by:

$$\rho(p, T) = \rho_0 \frac{p}{760} \frac{288.15}{T} \tag{14}$$

where:

$\rho_0$  is the density of dry air at Standard Temperature and Pressure, STP;  
chosen according to the International Standard Atmosphere ISA model,  
=1.225 [kg/m<sup>3</sup>] at 288.15 kelvin =15°C, 760 mm Hg.

The temperature and pressure are both functions of height above sea level. Taking a typical density of air at sea level of:

$$\rho = 1.225 \text{ [kg/m}^3\text{]},$$

the power flux at 760 mm of mercury and an absolute temperature of 288.15 kelvin or 15 °C becomes according to Eqn. 12:

$$\begin{aligned} P' &= \frac{1}{2} \rho V^3, \\ &= \frac{1}{2} \times 1.225 \times \frac{288.15 \times 760}{760 \times 288.15} V^3 \quad (15) \\ &= 0.6125 V^3 \left[ \frac{\text{Watts}}{\text{m}^2} \right]. \end{aligned}$$

Notice that there exist several definitions of STP. In the metric system of units, the standard reference conditions of temperature and pressure for expressing gas volumes is 0° Celsius or 273.15 kelvin and 101.325 kPa or 1 atm.

Many different other definitions of standard reference conditions exist throughout the world. For instance, standard reference conditions are also defined at 0 °C and 100 kPa or 1 bar, in contrast to 0 °C and 101.325 kPa or 1 atm. In Europe and South America a standard of 15 °C or 59 °F and 101.325 kPa or 14.696 psi is being used. The International Organization of Standardization, ISO, the USA Environmental Protection Agency, EPA, and the USA National Institute of Standards and Technology, NIST have more than one definition of STP. The USA National Bureau of Standards, NBS, has a standard for STP at 100 kPa or 1 bar, but with the temperature varying and usually specified separately, such as 20 °C and an absolute pressure of 101.325 kPa used by the USA EPA and NIST, if complete information is desired. Some standards specify the percent Relative Humidity, RH.

The International Standard Atmosphere, ISA model divides the atmosphere into layers with linear temperature distributions. At sea level, the standard gives a pressure of 101.325 kPa and a base temperature of 15 °C in the Troposphere at a base geometric height of 0 km with a lapse rate of – 6.5 °C/km. The ISO organization publishes the ISA as an international standard: ISO 2533:1975, with other organizations such as the International Civil Aviation Organization (ICAO) publish extensions under their own standards.

The ICAO International Standard Atmosphere, ISA standard conditions for zero density altitude are 0 meters (0 feet) altitude, 15 °C or 59 °F air temp, 1,013.25 mb or 29.921 in Hg pressure and 0 % relative humidity at the absolute zero dew point. The standard sea level air density is 1.225 kg/m<sup>3</sup> or 0.002378 slugs/ft<sup>3</sup>.

## **EXAMPLE**

The power flux through 1 square meter of wind at a speed of 5 m/s is:



$$\begin{aligned}
P' &= 0.6125V^3 \\
&= 0.6125 \times 5^3 \\
&= 76.56 \left[ \frac{\text{Watts}}{\text{m}^2} \right].
\end{aligned}$$

This is less than the power of one standard 100 Watt light bulb.

### EXAMPLE

At a wind speed of 10 m/s the power flux is:

$$\begin{aligned}
P' &= 0.6125 \times 10^3 \\
&= 612.5 \left[ \frac{\text{Watts}}{\text{m}^2} \right].
\end{aligned}$$

Or the equivalent of about 6 standard 100 Watt light bulbs.

This identifies wind power as a diffuse form of energy; a characteristic that is shared with solar energy, both requiring large collection areas to capture significant energy amounts. This affects the capital cost needed for the manufacture and construction of the wind and solar collection components.

The other important observation is that wind power increases rapidly with increased wind speed. Thus, overspeed protection devices such as hydraulic brakes must be used in conjunction with wind turbines to protect the turbine and the load under the circumstances of high wind speed.

The more general Eqn. 14 accounting for the air density should be used for a wind turbine that is located substantially above sea level and operating at higher temperatures.

The presence of wind turbines in large air masses modifies the local air pressure and speed.

### EFFECT OF DENSITY, COMPARISON TO HYDRAULIC SYSTEMS

Hydroelectric and wind power generation both extract energy from a flowing fluid, in one case it is water and the other air. We can write for the ratio of power extracted in the two cases:

$$\frac{P_{water}}{P_{air}} = \frac{\frac{1}{2} \rho_{water} S V^3}{\frac{1}{2} \rho_{air} S V^3} = \frac{\rho_{water}}{\rho_{air}} \quad (16)$$

for the same cross sectional area S and for the same speed V.

Taking the density of air as:

$$\begin{aligned}
\rho_{air} &= 1.225 \left[ \frac{\text{kg}}{\text{m}^3} \right] \text{ at } 288.15 \text{ kelvin } = 15^\circ\text{C}, 760 \text{ mm Hg} \\
&= 1.225 \left[ \frac{10^3 \text{ gm}}{10^6 \text{ cm}^3} \right] \\
&= 1.225 \times 10^{-3} \left[ \frac{\text{gm}}{\text{cm}^3} \right]
\end{aligned}$$

and the density of water as 1 gm / cm<sup>3</sup>, yields:

$$\begin{aligned}
P_{water} &= P_{air} \frac{\rho_{water}}{\rho_{air}} = \frac{1}{1.225 \times 10^{-3}} P_{air} \\
&= 816.3 P_{air}
\end{aligned} \tag{17}$$

This implies that extracting the diluted energy from the air requires the use of turbo-machinery that is about 800 times the dimensions of that used for the extraction of the water's more concentrated form of energy. The power extracted is proportional to the square of the linear dimension and the weight is proportional to the cube of the speed dimension; a property shared with airplane designs.

A wind machine, in addition "... works night and day under the most unfavorable climatic conditions, exposed such as it is to winds, gusts, variations of temperature, rain snow, hail, the rays of the sun, humidity, etc. "[1]

Regardless of the difficulties, the ageless established economic aspect of the availability of free fuel makes wind machines attractive, particularly on the small scale.

Most important, for remote locations that are inaccessible to the electric grids, it must be observed that if one has no water or no energy, and the issue is about some or none, no price is too high to pay.

## **POWER CURVE OF A WIND TURBINE**

The graph showing the variation of a turbine's power output as a function of wind speed is designated as a power curve. It shows the cut-in wind speed or the minimum at which the turbine is controlled to operate, the rated wind speed at which the turbine reaches its rated power, and the cut-out or shut down wind speed, at which the turbine shuts down to prevent damage.

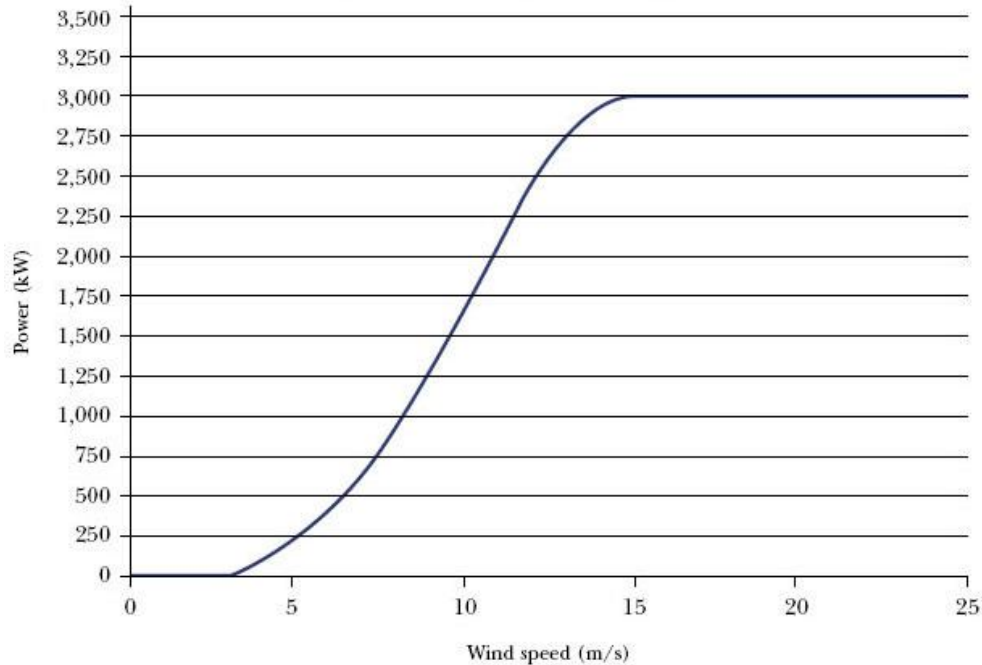


Figure 4. Power curve of the Vestas V90 3 MW rated power wind machine. The cut-in wind speed is about 3.5 m/s, and the cut-out wind speed is 25 m/s. Source: Vestas.

Long periods of operation above the rated power would damage the electrical generator and produce excessive mechanical vibrations and stresses.

As the wind speed increases past the turbine's rated speed, the control mechanism of the rotor limits the power drawn from the wind in order to keep the drive train torque constant.

The aerodynamic characteristics of stall-regulated machines results in a reduction in the power output at higher wind speeds rather than to produce a shut down. Pitch regulated machines, in comparison, produce a shutdown.

## POWER CONTROL

Since the power density  $W$  of the wind increases with the cube of the wind speed  $V$ , wind turbines must limit the power output in very high winds.

There exist two principal methods of accomplishing this, with pitch control on the blades or with fixed, stall-controlled blades.

Pitch-controlled blades are rotated as the wind speeds increase so as to limit the power output and, once the rated power is reached; a reasonably steady output can be achieved, subject to the control system response.

Stall-controlled rotors have fixed blades which gradually stall as the wind speed increases, thus limiting the power by passive means. This eliminates the need for a pitch control mechanism, but it is rarely possible to achieve constant power as wind speeds rise. Once peak output is reached the power tends to fall off with increasing wind speed, and so the wind energy capture may be less than that of a pitch controlled machine.

A typical power curve for a 1.65 MW turbine illustrates the concept of rated output. Most machines start to generate at a similar cut in speed at about 3 to 5 m/s and shut down in very high winds, generally around 20 to 25 m/s.

The merits of the two approaches are practically equal; resulting in almost the same number of existing pitch controlled and stall-controlled wind turbines.

## **PRACTICAL ENERGY PRODUCTION**

Energy yields do not increase continually with the cube of the wind speed, mainly because energy is discarded once the rated wind speed at about 15 m/s is reached.

The annual energy production from a typical wind turbine is around 2,100 MW.hr at a site where the wind speed is 5 m/s, 4,400 MW.hr at 7 m/s and 8,750 MW.hr at 8 m/s.

The implication is that placing an inexpensive wind turbine of limited performance on a good wind site is better than placing an expensive high performance wind turbine on a poor wind site.

Wind speeds around 5 m/s can be found inland on the five continents, but wind energy developers generally aim to find higher wind speeds.

Levels around 7 m/s are found in many European coastal regions, higher levels are found on many Greek Islands, in the California mountain passes, and on upland and coastal sites in the Caribbean, Ireland, Sweden, the United Kingdom, Spain, New Zealand and Antarctica.

Wind speed is the primary determinant of electricity cost, because of the way it practically influences the power and hence the energy yield, for the same rotor swept area:

$$P_2 = P_1 \left( \frac{V_2}{V_1} \right)^3 \quad (18)$$

Developments on sites with an average wind speed of 8 m/s will yield 4 times the power for the same turbine swept area than a 5 m/s site:

$$P_2 = P_1 \left( \frac{8}{5} \right)^3 = 4.1 P_1$$

Offshore wind speeds are generally higher than those onshore, favoring offshore sites.

## **DISCUSSION**

The utilization of wind power offers three categories of wind machines:

### **1. Small, individual scale:**

Simple machines of about 2-5 kW in rated power capacity associated with full energy storage in battery banks and load distributors for individual premises. A load

distributor is a device that ensures that when the wind blows, the extracted energy is channeled into useful purposes such as heating dwellings, barns and greenhouses, or electrically pumping water; in addition to charging and storage in chemical storage batteries. An alternative to battery storage is wind turbine pumping of water to a higher elevation, then releasing the accumulated potential energy into kinetic energy to rotate a water wheel or turbine when the energy is needed.

## 2. Medium, community scale:

Elaborate machines with capacity in the 5-100 kW range for small communities feeding into small local electrical networks backed up with a diesel generator.



Figure 5. The Vestas V90 3MW rated power wind turbine is an example of the economy of scale characteristic of utility scale large wind turbines. Source: Vestas.

## 3. Large, utility scale:

Sophisticated utility scale machines at the MW level feeding into main electrical supply grid networks.

The trend towards using large turbine is encouraged by their inherent economy of scale. Since the power production, for the same wind speed, is proportional to the rotor swept area:

$$P_2 = P_1 \frac{S_2}{S_1} = P_1 \frac{\pi R_2^2}{\pi R_1^2} = P_1 \left( \frac{R_2}{R_1} \right)^2 \quad (19)$$

Doubling the rotor radius from  $R_1$  to  $2R_1$  leads to a power increase of:

$$P_2 = P_1 \left( \frac{2R_1}{R_1} \right)^2 = 4P_1$$

Thus an increase in the rotor radius by a factor of 2 leads to a power increase by a factor of 4.

However, large wind turbines possess two limitations:

1. The first is that, being massive, they need to be shut down at stormy wind speeds higher than about 25 m/sec to prevent their structural damage.
2. The second is that they have a relatively high cut-in wind speed of 3.5-6 m/sec. Thus they are limited to the extraction of a fraction of the wind speed spectrum.



Figure7. “Skimming” the wind for pumping water was practiced using about 10,000 sail-driven small wind turbines during the summer on the Lassithion plateau, Crete [2].

Photo: George Deliyannakis.

Small wind turbines do not suffer these two limitations. They have a lower cut-in wind speed at 2-3 m/sec, and they can withstand higher wind gust of 25-30 m/sec and higher. This means that they can extract a larger fraction of the wind spectrum than large machines.

This allows us to suggest the possibility of a system of “skimming” the wind in a given extended land location such as a desert or mountainous region, by covering it with a distributed system or a blanket of a large number of small economic wind turbines exploiting both its lower wind speeds as well as its wind gusts; a feat that the larger size turbines are incapable of.

The efficiency or performance coefficient of such a large number of small turbines becomes irrelevant, considering that the wind is free; thus simple and inexpensive locally manufactured designs can be used. The analogy here is the use of a distributed system of a large number of cheap simple personal computers rather than a limited number of localized expensive and sophisticated supercomputers.



Figure 8. Small water-pumping wind turbine in Eastern Crete with its sail furled during the rainy winter season [2].

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## EXERCISES

1. A wind turbine has a rated power of 3 MW at a site with an average wind speed of 15 m/s.  
Estimate:
  - a. Its maximum possible yearly energy output in MW.hrs,
  - b. Its yearly energy production in MW.hrs at sites with the following average wind speeds:
    - i. 5 m/s,
    - ii. 8 m/s,
    - iii. 10 m/s.
2. To develop maximum power, a wind generator must be mounted as high as possible. Several authors have suggested the following simple law of variation of wind speed with height:

$$\frac{V}{V_0} = \left( \frac{H}{H_0} \right)^n$$

where  $H_0$  is the observed speed at  $H_0$  meters above ground, and  $V$  is the wind speed at altitude  $H$ .

The value of  $H_0$  is usually given at 10 m height, and the coefficient  $n$  takes values over the range 0.1 – 0.4.

The wind speed at 20 meters height at the Eiffel Tower is about 2 m/s, and it is about 7-8 m/s at 300 meters above ground.

What range of values of the coefficient  $n$  best fits the Eiffel Tower situation?