Chapter 1

RADIOACTIVE TRANSFORMATIONS THEORY, THE WEAK FORCE

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1.1 INTRODUCTION

At any given moment thousands of rays of radiation are crisscrossing the human body. These rays are invisible, and are just part of nature. It is important that we all become literate about radiation, to alleviate unsubstantiated fears and to be able to protect our coworkers, families and ourselves when undesirable situations involving radiation arise, such as the accumulation of Radon^{222} and its daughter products in some dwellings.

The quest by the middle age alchemists for the philosopher's stone that has the power to transmute one element into another had been the obsession of many great thinkers throughout history including Sir Isaac Newton, Robert Boyle and John Locke. This dream of the ancient alchemists was achieved in 1919 at Manchester University in the UK.

A student of New Zealand born scientist and Nobel Prize winner Ernest Rutherford noticed that when radioactive materials such as radium were placed in a sealed container of air, small amounts of hydrogen, which does not exist in ordinary air, began to mysteriously appear. Ernest Rutherford explained the occurrence that the powerful radioactive alpha particle from the decay of radium interacted with nitrogen, which constitutes about ¾ of the air we breathe and transmuted into two other gases: hydrogen and an isotope of oxygen, through the nuclear reactions:

$$
\begin{aligned}
&{}_{88}Ra^{226} \rightarrow {}_{86}Rn^{222} + {}_{2}He^{4} \\
&{}_{2}He^{4} + {}_{7}N^{14} \rightarrow {}_{1}H^{1} + {}_{8}O^{17} \\
&\text{---------------------------------------} \\
&{}_{88}Ra^{226} + {}_{7}N^{14} \rightarrow {}_{1}H^{1} + {}_{8}O^{17} + {}_{86}Rn^{222}
\end{aligned}
\qquad (1)
$$

The alpha particles being produced by the radium were acting as catalysts in an overall reaction where the radium and nitrogen nuclei were turning into radon and oxygen while knocking out single protons which, by combination with electrons became hydrogen atoms then hydrogen gas molecules.

Nowadays the transmutation of an element into another through nuclear reactions is a routine process, with this modern nuclear alchemy playing a major role in the way we produce and use energy.

1.2 MODES OF RADIOACTIVE DECAY

Some nuclei in nature, and some artificially created, attempt at reaching a stable configuration in the nucleus through emitting the excess particles or electromagnetic radiation. These nuclear transformations are called radioactive transformation, or just radioactivity.

The main modes of radioactive transformations are:

1. **Negative beta decay:** These are electron emissions from the nucleus. They occur primarily in neutron rich nuclei in an attempt at reaching stability by increasing the number of protons in the nucleus.
2. **Positron decay:** Here positive electrons, the anti matter of negative electrons are emitted by proton rich nuclei in an attempt at reaching stability by decreasing the number of protons in the nucleus.
3. **Alpha decay:** This is the emission of a whole helium nucleus from the parent nuclide. This mode of radioactivity primarily occurs among the heavy nuclides such as $_{92}\text{Uranium}^{238}$.
4. **Gamma decay:** This is an emission of electromagnetic radiation of very short wave length from excited nuclei on their way to reaching the ground state. It can occur by itself, but normally accompanies beta decay.
5. **Orbital electron capture:** Occurs in proton rich nuclides, where an inner shell electron is grabbed by the nucleus, with the emission of characteristic x rays from the ensuing electron transition.
6. **Delayed Radiations:** Can involve neutrons, protons alphas and gamma emissions. Delayed neutron emission is notable in fission products and affects the control of fission reactors.
7. **Isomeric Transitions:** Occurs when gamma rays are emitted for an excited nucleus to reach its ground state.
8. **Internal conversion:** Involves the direct transfer of energy from the nucleus to one of the orbital electrons, and the electron is ejected from the atom.
9. **Spontaneous fission:** Some heavy nuclei decay in a process where the nucleus breaks up into two intermediate mass fragments and several neutrons. It occurs in with nuclei with a mass number $A > 230$.
10. **Double beta decay:** A rare radioactive event observed for Mo^{92} and Mo^{100}.
11. **Cluster decay:** Has been observed in several heavy nuclides where clusters of C^{12}, C^{14}, O^{20}, Ne^{20}, Mg^{28}, or Si^{32} are emitted.

1.3 RADIOACTIVE DECAY LAW

HEURISTIC Approach

Consider an initial number of radioactive nuclides at time $t = 0$ as N_0. These nuclei would be undergoing radioactive transformations and the initial number of nuclei is going to decrease over time. If we consider the time at which the initial number is decreased by one half to $N_0/2$, we can designate this time as the half life $T_{1/2}$. These nuclei would continue decaying to ¼ their initial value after 2 half lives, to 1/8 of their initial value after 3 half lives, and so on. In general, after n half lives, these nuclei would have decayed to $(1/2)^n$ their initial value N_0 as shown in Table 1 below.

Table 1. Number of radioactive nuclei present after n half lives.

Number of half-lives	Elapsed time	Number of nuclei present
0	0	N_0
1	1 $T_{1/2}$	$N_0/2$
2	2 $T_{1/2}$	$N_0/4$
3	3 $T_{1/2}$	$N_0/8$
---	---	---
n	n $T_{1/2}$	$N_0/2^n$

Using mathematical induction the number of nuclei present after n half-lives is given by:

$$N(n) = N_0\left(\frac{1}{2}\right)^n \tag{2}$$

Since the time elapsed t is equal to the number of half-lives:

$$t = n.T_{1/2}$$

from which:

$$n = \frac{t}{T_{1/2}},$$

Equation 2 can thus be written as a function of time by substituting for n as;

$$N(t) = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} \tag{3}$$

This law of radioactive decay has a different form that can be derived based on differential calculus considerations.

EXPONENTIAL DECAY LAW

Let the change during a time period dt in the number of radioactive nuclei present be dN(t). The change dN(t) is both proportional to the number of nuclei present N(t) and the time interval dt:

$$dN(t)\ \alpha_{\ell}\ -N(t)dt \tag{4}$$

The negative sign accounts for the fact that the radioactive nuclei are decreasing in number as a function of time. The proportionality sign can be replaced by an equality sign if we add a decay constant λ to Eqn. 4 as:

$$dN(t) = -\lambda N(t)dt \tag{5}$$

To determine N as a function of time, we separate the variables in Eqn. 5:

$$\frac{dN(t)}{N(t)} = -\lambda dt$$

This equation can be integrated from the initial time t = 0 to any time t using limit integration:

$$\int_{N_0}^{N(t)} \frac{dN(t)}{N(t)} = -\lambda \int_0^t dt$$

Integrating yields:

$$\ln N(t)\Big|_{N_0}^{N(t)} = -\lambda t \;\Big|_0^t$$

Substituting the upper and lower limits, we get:

$$\ln N(t) - \ln N_0 = \ln \frac{N(t)}{N_0} = -\lambda t$$

Taking the exponential of both sides yields:

$$e^{\ln \frac{N(t)}{N_0}} = \frac{N(t)}{N_0} = e^{-\lambda t}$$

This yields a negative exponential process described by the radioactive decay law with N(t) being the number of nuclei present after a certain time t:

$$N(t) = N_0 e^{-\lambda t} \tag{6}$$

where: N_0 is the initial number of nuclei present at time t = 0
λ is the radioactive decay constant.

1.4 HALF-LIFE AND MEAN LIFE

DEFINITION OF HALF-LIFE

The half life $T_{1/2}$ is the time at which a radioactive isotope's number of nuclei at time t, N(t), has decayed to one half its initial number of nuclei $N_0/2$. Expressing this fact in the radioactive decay law above, one can write:

$$N(T_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

Canceling the N_0 term on both sides of the equation we get:

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

To eliminate the exponential we take the natural logarithm of both sides of the equation, yielding:

$$\ln 1 - \ln 2 = -\lambda\, T_{1/2}$$

Substituting ln 1 = 0, we can express the half-life in terms of the decay constant as:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931}{\lambda} \qquad (7)$$

Similarly, we can express the decay constant in terms of the half-life as:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.6931}{T_{1/2}} \qquad (7)'$$

This suggests another form of the radioactive decay law:

$$N(t) = N_0 e^{-\frac{\ln 2}{T_{1/2}} t} = N_0 e^{-\frac{0.6931}{T_{1/2}} t} \qquad (8)$$

EQUIVALENCE OF THE TWO FORMS OF THE RADIOACTIVE DECAY LAW

One can express the radioactive decay law in terms of the number n of half lives elapsed:

$$n = \frac{t}{T_{1/2}},$$

in the form:

$$N(n) = N_0 e^{-n \ln 2}$$

We note that:

$$e^{-n \ln 2} = \frac{1}{e^{n \ln 2}} = \frac{1}{e^{\ln 2^n}} = \frac{1}{2^n},$$

then:

$$N(n) = N_0 \left(\frac{1}{2} \right)^n,$$

which is Eqn. 2, and proves that the two forms of the radioactive decay law are equivalent.

A radioactive isotope, according to the radioactive decay law in either of its forms, has substantially decayed after a few half lives. For instance, after seven half lives only:

$$\left(\frac{1}{2} \right)^7 = \frac{1}{128}$$

or less than 1 percent of the original amount remains. One can construct a simple Table 2 showing the fraction of a radioactive isotope remaining after n half lives.

A rule of thumb is that after ten half lives; only 1/1000 of the original nuclei are remaining. After twenty half lives, only 1 millionth of the original nuclei remain. And after thirty half lives; only one billionth of the original radioactive nuclei would have decayed. Thus it does not take a large number of half lives for a radioactive sample to decay.

Table 2. Fraction of a Radioactive Isotope remaining after n half lives.

Number of half lives (n)	Fraction remaining
1	½
2	¼
3	1/8
4	1/16
5	1/32
6	1/64
7	1/128
8	1/256
9	1/512
10	1/1024

MEAN LIFETIME

The process of radioactive transformation is a random process. The mean lifetime or average life expectancy is the mathematical expectation of the time that it takes a radio nuclide to decay over the law of radioactive decay as a probability density function.

$$\tau = \frac{\int_0^\infty t dN(t)}{\int_0^\infty dN(t)} = \frac{\int_0^\infty t\lambda N_0 e^{-\lambda t} dt}{\int_0^\infty \lambda N_0 e^{-\lambda t} dt} = \frac{\int_0^\infty te^{-\lambda t} dt}{\int_0^\infty e^{-\lambda t} dt} = \frac{\int_0^\infty te^{-\lambda t} dt}{\left.\frac{e^{-\lambda t}}{-\lambda}\right|_0^\infty} = \lambda \int_0^\infty te^{-\lambda t} dt$$

Integrating by parts:

$$\begin{aligned}\tau &= \lambda \int_0^\infty te^{-\lambda t} dt = \frac{\lambda}{-\lambda} \int_0^\infty t de^{-\lambda t} = \int_\infty^0 t de^{-\lambda t} \\ &= te^{-\lambda t}\Big|_\infty^0 - \int_\infty^0 e^{-\lambda t} dt = -\int_\infty^0 e^{-\lambda t} dt = -\frac{1}{-\lambda} e^{-\lambda t}\Big|_\infty^0 \\ &= \frac{1}{\lambda}\end{aligned} \qquad (9)$$

Thus, the mean life is simply the inverse of the decay constant.

The law of radioactive transformations can be expressed in terms of the mean life in another form as:

$$N(t) = N_0 e^{-\frac{t}{\tau}} \qquad (10)$$

DECAY CURVES

The transformation of tritium T^3 into the He^3 isotope is governed by its radioactive decay equation with N(t) being the number of nuclei present after a certain time t:

$$\frac{N(t)}{N_0} = e^{-\lambda t} = e^{-\frac{\ln 2}{12.33} t}$$

where: N_0 is the initial number of nuclei present at time t = 0

λ is the decay constant = $\frac{\ln 2}{T_{1/2}} = \frac{0.6931}{T_{1/2}}$.

$T_{1/2}$ is the half life for tritium = 12.33 years.

An American National Standards Institute (ANSI) Fortran-90 (f-90) procedure that can display the decay features of the tritium isotopes is listed here:

```
!     Decay Curve generation for Tritium
```

```
!       N(t)=No*exp(-lambda*t)
!       lambda=decay constant=  ln 2 / T
!       T=half-life
!       Program written in ANSI Fortran
!       Digital Visual Fortran Compiler
!       Procedure saves output to file:output1
!       This output file can be exported to a plotting routine
!       M. Ragheb, University of Illinois
        program decay
        real x, lambda
!       This half life is for the tritium 1T3 nucleus
        real :: T = 12.33
        integer :: steps=100
        real ratio(101), xtime(101)
!       Calculate decay constant
        x = log(2.0)
        lambda = x/T
        write(*,*) x, lambda
!       Open output file
        open(10,file='output1')
!       Calculate ratio N(t)/No
        steps = steps + 1
        do i =  1, steps
                xtime(i) = i - 1
                ratio(i) = exp (- lambda*xtime(i))
!       Write results on output file
                write(10,*) xtime(i), ratio(i)
!       Display results on screen
                write(*,*) xtime(i), ratio(i)
!               pause
        end do
        end
```

The output file was plotted in Fig. 1 using the Excel Plotting Package where the ratio $N(t)/N_0$ is shown, and shows the rapid decay of tritium as a function of time.

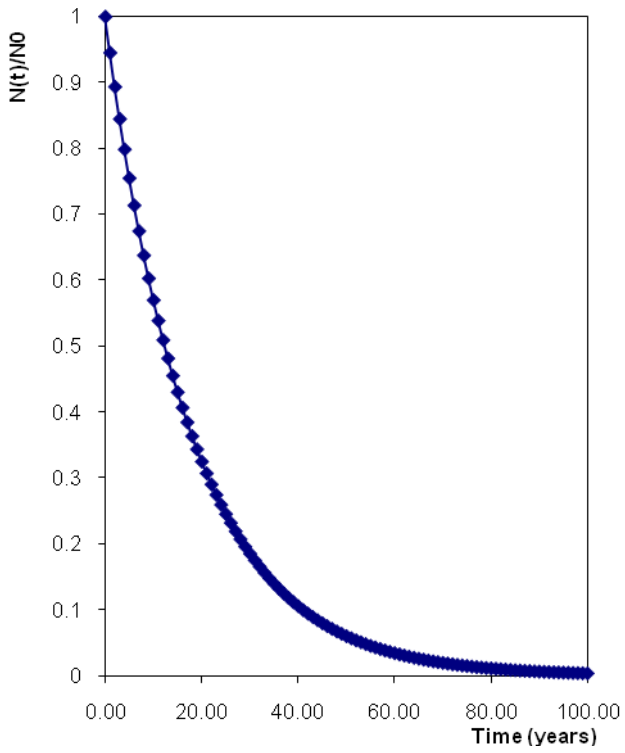


Figure 1. Decay curve for the tritium isotope.

1.5 ACTIVITY

The measurement of the intensity of radioactive transformations is possible if we use the rate of radioactive transformations from Eqn. 6 as:

$$\frac{dN(t)}{dt} = \frac{d}{dt}(N_0 e^{-\lambda t}) = -\lambda N(t) \tag{11}$$

instead of just the number of nuclei present. This is achieved in terms of the radiological quantity designated as "Activity." It is defined as the positive value or magnitude of the product of the radioactive decay constant and the number of radioactive nuclei present at any time t:

$$A(t) = \left|\frac{dN(t)}{dt}\right| = \left|-\lambda N(t)\right| = \lambda N(t) \tag{12}$$

This can also be written in the form:

$$A(t) = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t} \tag{13}$$

Usually it is estimated at time t = 0 as:

$$A_0 = \lambda N_0 \tag{13'}$$

In terms of the mean lifetime, it can be expressed as:

$$A(t) = A_0 e^{-\frac{t}{\tau}} \tag{14}$$

or:

$$A(t) = \frac{N(t)}{\tau} \tag{15}$$

$$A_0 = \frac{N_0}{\tau} \tag{15'}$$

It is equal to the number of transformations per unit time. The unit used in the Système International (SI) system of units is:

1 Becquerel = 1 Bq = 1 [Transformation/sec].

In the conventional system of units, the unit used is the Curie, where:

1 Curie = 1 Ci = $3.7\ 10^{10}$ Bq,

which is the amount of activity of 1 gm of the $_{88}$Radium^{226} isotope. Smaller units are used:

1 milliCurie = 1 mCi = 10^{-3} Ci,
1 microCurie = 1 μCi = 10^{-6} Ci.

Half lives vary widely from one radioactive isotope to another. Excited states of some nuclei decay with half lives in a range from a thousandth to a trillionth of a second. On the other hand, the most common naturally occurring isotope of Uranium, $_{92}U^{238}$, has a half-life of 4.468 billion years. Similarly, the naturally occurring potassium isotope $_{19}K^{40}$ exists in the human body at an abundance of 0.01 percent and has a half life of 1.27 billion years.

Tritium, $_1T^3$, a man made isotope of hydrogen, does not exist in nature, except for trace amounts from cosmic ray interaction with hydrogen in the atmosphere, and has a half life of 12.33 years. For tritium to decrease to 1/1000 of its initial amount, would require 10 half lives or

about 123 years. On the other hand, only a few years are needed for a significant decrease in the amount of tritium initially present. As a consequence, the tritium used in boosted nuclear fission devices has to be replenished every few years for a viable nuclear weapons arsenal.

Interestingly, for $_{92}U^{238}$ to decrease to 1/1000 of its initial amount would require 44.68 billion years. This is about three times the age of the universe at about 15 billion years. It will be there for a long time. On the other hand $_{94}Pu^{239}$, the man made fissile isotope, has a half life of 24,110 years, and decays into the naturally occurring $_{92}U^{235}$ isotope through alpha particle emission and eventually into a stable lead isotope. For this reason, primordial $_{94}Pu^{239}$ is not found in nature any more. Trace amounts of it can be found in uranium ores as a result of spontaneous fission neutrons capture in U^{238}.

The isotope of strontium $_{38}Sr^{90}$, a fission product, has a half life of 29 years, so it would take a life span for it to decay significantly. Thus if ingested in the human body, where it mimics calcium, it seeks the bone system, and remains there for practically a lifetime.

An isotope used in medical studies, $_{8}O^{15}$, has only a 2 minutes half life. A patient who is injected with this isotope will have only 1/1000 of the original radioactive dose of the isotope present after 20 minutes. After one hour, amounting to thirty half lives, only one billionth the original amount of the radioactivity remains. This is at the core of the beneficial uses of radioisotopes in nuclear medicine applications.

1.6 SPECIFIC ACTIVITY, ACTIVITY DENSITY

When the activity is estimated per unit mass of a solid material, it is designated as specific activity. The most commonly used units in this case are:

1 Curie/gram = 1 Ci /gm,
1 Becquerel/gram = 1 Bq / gm

When a liquid is under consideration, the activity density rather than specific activity is used, such as:

1 Curie/liter = 1 Ci /l,
1 Becquerel/cubic centimeter = 1 Bq/cm^3.

1.7 DETERMINATION OF HALF LIFE

The half life, and the decay constant can be determined experimentally. Taking the natural logarithm of the radioactive transformation law, expressed in terms of the activity of a sample:

$$\ln \frac{A(t)}{A_0} = \ln e^{-\lambda t} = -\lambda t$$

This appears to be an equation of a straight line with a negative slope of $m = \lambda$.

$$\ln A(t) = \ln A_0 - \lambda t$$
$$y(t) = y_0 - mt \qquad (16)$$

If the logarithm of the measured activity is plotted against the time t, a straight line should result with a slope of $-\lambda$, which itself is equal to ln 2 / $T_{1/2}$, allowing for the experimental determination of the half life.

1.8 PRODUCTION OF RADIO NUCLIDES

Radioactive isotopes can be produced by bombardment with charged particles such as protons or helium ions in particle accelerators. However, the most efficient way is to produce them with the bombardment with neutrons, since they do not have to overcome the Coulomb barrier of the nucleus like charged particles have to do. Nuclear reactors being a copious source of neutrons have been used for the production of radioactive isotopes through the neutron irradiation of otherwise stable nuclides.

Assuming that the neutron bombardment hardly affects the original material; a good assumption in high flux reactors, the net rate of change of the number of radioisotopes present in a reactor will be equal to the production rate (Q) minus the decay rate of the isotope or:

$$\frac{dN(t)}{dt} = Q - \lambda N(t) \qquad (17)$$

Rearranging:

$$\frac{dN(t)}{dt} + \lambda N(t) = Q$$

Multiplying both sides by an integrating factor $e^{\lambda t}$, converts the left hand side into a total differential

$$e^{\lambda t}\frac{dN(t)}{dt} + e^{\lambda t}\lambda N(t) = Qe^{\lambda t}$$

$$\frac{d[N(t)e^{\lambda t}]}{dt} = Qe^{\lambda t}$$
$$d[N(t)e^{\lambda t}] = Qe^{\lambda t}dt$$

Integrating both sides yields:

$$\int_{N_0}^{N(t)} d[N(t)e^{\lambda t}] = \int_0^t Qe^{\lambda t}dt$$

$$N(t)e^{\lambda t} - N_0 = \frac{Q}{\lambda}(e^{\lambda t} - 1)$$

Multiplying both sides by $e^{-\lambda t}$ results in:

$$N(t) = N_0 e^{-\lambda t} + \frac{Q}{\lambda}(1 - e^{-\lambda t}) \tag{18}$$

If the initial number of isotopes is zero, $N_0 = 0$:

$$N(t) = \frac{Q}{\lambda}(1 - e^{-\lambda t}) \tag{19}$$

Written in terms of the activity generated, we get:

$$A(t) = \lambda N(t) = Q(1 - e^{-\lambda t}) \tag{20}$$

This equation describes a process by which the isotope builds up to a saturation value at $t = \infty$ of:

$$A_\infty = Q \tag{21}$$

It is worthwhile to bombard only for a period of 2 to 3 half lives, since 3/4 to 7/8 of the maximum number of nuclei (Q / λ) is then produced. Irradiating the isotope for a longer time becomes uneconomical since the cost of the irradiation could become prohibitive.

Upon stopping the irradiation, the radioactive isotope decays according to its own half-life as shown in Fig. 2:

$$A(t) = Q(1 - e^{-\lambda t_s})\, e^{-\lambda (t - t_s)} \tag{22}$$

where t_s is the time at which irradiation has stopped.

The last equation can be rewritten as:

$$A(t) = Q(e^{\lambda t_s} - 1)e^{-\lambda t} \tag{22}'$$

EXAMPLE

As an example, let us consider the production of the isotope $_{25}Mn^{56}$ from $_{25}Mn^{55}$. The latter has a natural abundance of 100 percent and is the only naturally occurring isotope of manganese. It can be placed in a nuclear reactor where it can undergo 10^{10} (n, γ) reactions per second. The reaction is:

$$_{25}Mn^{55} + {}_0n^1 \rightarrow {}_{25}Mn^{56} + \gamma$$

The formed isotope is radioactive and decays with a half life of 2.58 hours into a stable iron isotope, with the emission of a negative electron and an antineutrino:

$$_{25}Mn^{56} \rightarrow {}_{26}Fe^{56} + {}_{-1}e^{0} + \nu*$$

The decay constant can be calculated as:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.6931}{2.58} = 0.269[hr]^{-1}$$

One can calculate the activity reached after 5 hours as:

$$A(5\ hrs) = 0 + 10^{10}\ (\ 1 - e^{-0.269 \times 5})$$
$$= 7.39 \times 10^{9}\ [Bq]$$
$$= 7.39 \times 10^{9} / (3.71 \times 10^{10}) = 0.2\ [Ci]$$

Here we considered that there was a zero value of the generated isotope at the time of initial irradiation.

1.9 PROCEDURE FOR THE ESTIMATION OF THE PRODUCTION OF AN ISOTOPE

The following procedure can be used to estimate the growth of the activity for the production of an isotope in a nuclear reactor, and its subsequent decay according to Eqns. 20 and 22.

```
!       Activity buildup curve for the production of an isotope:
!       A(t)=Q*(1-exp(-lambda*t))
!       Followed by decay after end of irradiation
!       A(t)=Q*(1-exp(-lambda*tira))*exp(-lambda*t)
!       tira = irradiation time
!       lambda = decay constant = ln 2 / T
!       T=half-life
!       Program saves output to file:output1
!       This output file can be exported to a plotting routiine

        program isotope_production
        real x, lambda
!       This half life is for the 25Mn56 nucleus in hours
!       It is formed through neutron irradiation from 25Mn55
!       It decays to 26Fe56 through negative beta emission
        real :: T = 2.54
!       Q is production rate
        real :: Q = 1.0E+10
!       Conversion ratio from Becquerels to Curies
        real :: C =3.71e+10
!       Irradiation time: 10 hours, decay time: 10 hours
        integer :: steps=20
        real activity(51), xtime(51)
```

```
!       Calculate decay constant
        x = log(2.0)
        lambda = x/T
        write(*,*) x, lambda
!       Open output file for plotting in Excel
        open(10,file='output1.xls')
!       Calculate ratio activity in Curies
        steps = steps/2.0
!       Irradiation time
        do i =  1, steps
                xtime(i) = i - 1
                activity(i) = Q*(1-exp (- lambda*xtime(i)))/C
!       Write results on output file
                write(10,*) xtime(i), activity(i)
!       Display results on screen
                write(*,*) xtime(i), activity(i)
!       Store irradiation activity in Curies
                QQ=activity(i)
        end do
        jj=steps
!       Decay time
        do i =  1, steps+1
                xtime(i) = jj+i-1
                ti=i
                activity(i) = QQ*exp (- lambda*ti)
!       Write results on output file
                write(10,*) xtime(i), activity(i)
!       Display results on screen
                write(*,*) xtime(i), activity(i)
        end do
        end
```

Figure 2 shows the growth of the activity for the irradiation of the manganese isotope $_{25}Mn^{55}$ isotope for 10 hours to produce $_{25}Mn^{56}$ followed by 10 hours of its beta decay into $_{26}Fe^{56}$.

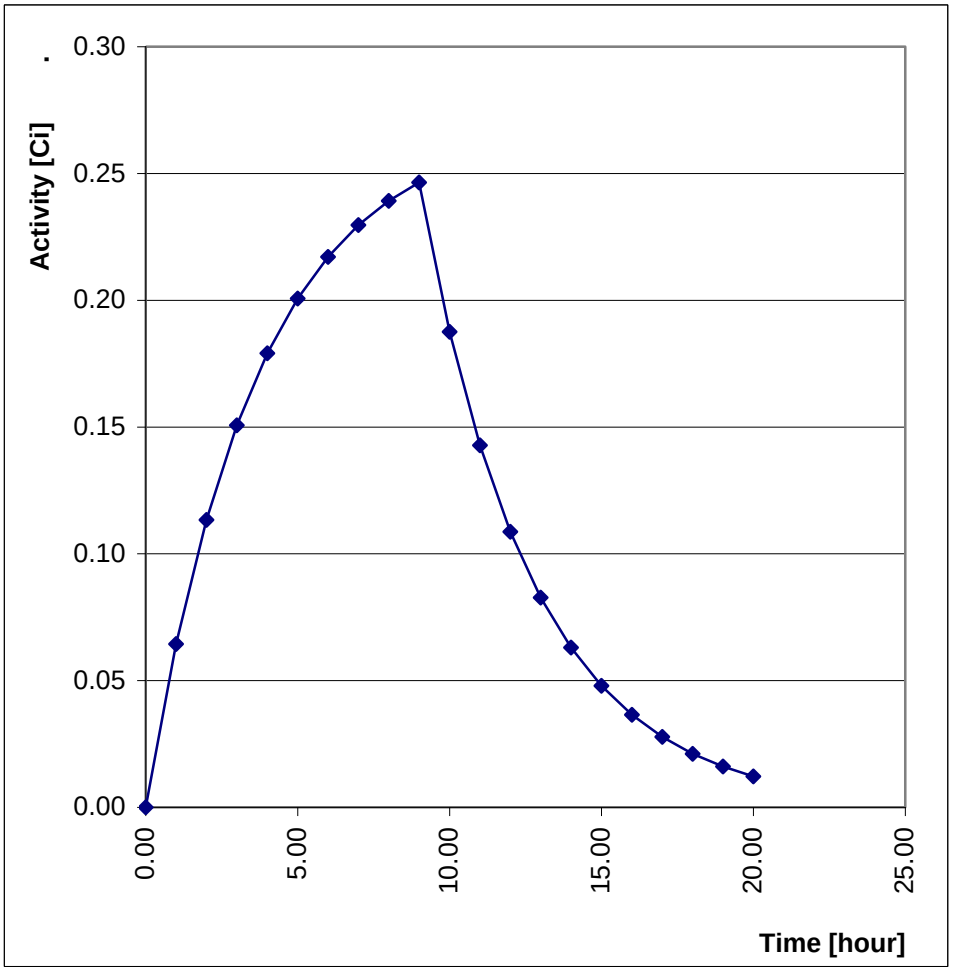


Figure 2. Growth of an irradiated isotope in a nuclear reactor, and its decay after the stoppage of irradiation.

1.10 RADIOISOTOPIC APPLICATIONS

Radioisotopes have wide usage in consumer products, industry, medicine, biology and scientific research. Most people are unaware that they are safely used in a wide variety of applications. Some of these peaceful as well as military applications can be listed in Table 3.

Table 3. Origin and Civilian and Military uses of Radioactive Isotopes.

Radioactive Isotope	Half Life $T_{1/2}$	Usage
$_{95}Americium^{241}$	432 y	Smoke detectors for homes and businesses Measuring levels of toxic lead in dried paint samples Online thickness gauges to ensure uniform thickness in rolling processes like paper, steel, Aluminum, paper, and plastic production Oil wells logging
$_{48}Cadmium^{109}$	453 d	Analysis of metal alloys for checking stock, sorting scrap
$_{20}Calcium^{47}$	4.54 d	Biomedical research for the study of cell function and bone Formation in mammals

$_{98}$Californium^{252}	2.64 y	Its neutron emission used in the in airports for the inspection of airline luggage for hidden explosives Gauging the moisture content of soils in road construction and building industries Measuring the moisture content of materials stored in silos
$_{6}$Carbon^{14}	5730 y	Formed by cosmic rays neutrons bombardment of $_{7}N^{14}$ in the upper atmosphere Archaeological dating Research ensuring that new drugs are metabolized without forming harmful by-products
$_{55}$Cesium^{137}	30.17 y	Its gamma rays emission is used in the treatment of cancers Sterilization of medical products and food products against harmful pathogens such as E. Coli 0157:H7, Listeria, Salmonella, and Campylobacter Measuring correct patient dosages of radioactive pharmaceuticals Measuring and controlling liquid flow in oil pipelines Testing oil wells for sand blockages Height gauges for fill level for packages and containers of food, drugs and other products
$_{24}$Chromium^{51}	27.71 d	Research in red blood cells survival studies
$_{27}$Cobalt^{57}	271 d	In Nuclear Medicine for the interpretation of diagnosis scans of patients' organs, and for the diagnosis of pernicious anemia
$_{27}$Cobalt^{60}	5.27 y	Its gamma rays emissions used in the sterilization of surgical instruments and medical products Treatment of cancers Meat, poultry, fruits and spice products sterilization against harmful organisms Improving the safety and reliability of industrial fuel oil burners
$_{29}$Copper^{67}	61.7 h	Attached to monoclonal antibodies which seek cancer cells in the body to destroy them through radioactive emissions
$_{96}$Curium^{244}	18.11 y	Analysis of materials excavated from pits slurries from drilling operations in mining
$_{53}$Iodine^{123}	13.2 h	Diagnosis of thyroid gland disorders such as Grave's syndrome
$_{53}$Iodine^{129}	$1.59x10^{7}$ y	Check some radioactivity counters at in vitro diagnostic testing laboratories
$_{53}$Iodine^{131}	8.041 d	Formed as a fission product in the fission process Released from nuclear explosions and postulated reactor accidents Diagnosis and treatment of thyroid cancer nodules in Grave's syndrome
$_{77}$Iridium^{192}	74.2 d	Nondestructive testing the integrity of pipelines welds, boilers and aircraft parts
$_{26}$Iron^{55}	2.7 y	Analyzing electroplating solutions
$_{36}$Krypton^{85}	10.72 d	Used in indicator lights in electrical appliances such as cloth washers and dryers, stereos and coffee makers Measurement of dust and pollutant levels In thickness gauges in the manufacturing of thin plastic and sheet metal, rubber, textiles and paper
$_{28}$Nickel^{63}	100 y	Detection of explosive materials Voltage regulators and current surge protectors in electronic devices
$_{15}$Phosphorus^{32}	14.28 d	Used in molecular biology and genetics research
$_{94}$Plutonium^{238}	87.74 y	Its alpha emissions used as a heat source and through thermionic conversion as an electrical source in deep space probes and crafts Thermionic electrical source in imbedded heart pacers
$_{94}$Plutonium^{239}	$2.411x10^{4}$ y	Fuel for future breeder fission reactors Fission nuclear weapons devices
$_{84}$Polonium^{210}	138.38 d	Through its alpha particles emissions, elimination of static charges in the manufacturing of photographic film and records
$_{61}$Promethium^{147}	13.6 m	Used in electrical blankets thermostats Gauging the thickness of thin plastics,thin sheet metal, rubber, textiles and paper
$_{88}$Radium^{226}	1600 y	Daughter nuclide in the decay chain of $_{92}U^{238}$ Discovered by Marie and Pierre Curie

		Enhances the effectiveness of lightning rods
$_{86}$Radon^{222}	3.82 d	Daughter nuclide in the decay chain of $_{92}U^{238}$ Formed from the decay of $_{88}Ra^{226}$ Concentrates in overly insulated homes. Health hazard in home construction, uranium mining, and cigarette smoking.
$_{34}$Selenium^{75}	120 d	Used in protein studies in life sciences research
$_{11}$Sodium^{24}	15.02 h	Location of leaks in industrial pipelines Oil well logging studies
$_{38}$Strontium^{85}	65.2 d	Fission product under calcium in the periodic table of the elements Constituent of fallout from nuclear weapons testing Studies of bone formation and metabolism
$_{43}$Technetium^{99m}	6.02 h	Diagnostic studies in nuclear medicine including brain, bone, liver, spleen and kidney imaging and for blood flow studies
$_{81}$Thallium^{204}	3.77 y	Measurement of dust and pollutant levels on filter paper Thickness gauges in plastics, sheet metal, rubber, textiles and paper manufacturing
$_{90}$Thorium^{229}	7340 y	Used in making fluorescent lights last longer
$_{90}$Thorium^{230}	$7.7x10^4$ y	Used to breed $_{92}U^{233}$ as afissile fuel in thermal fission breeder reactors As thoriated tungsten, used in electric arc welding rods in the construction, aircraft, petrochemical, and food processing equipment industries to produce easier starting, enhanced arc stability and reduced metal contamination
$_{1}$Tritium^{3}	12.33 y	Life Science and drug metabolism in new drugs studies Self luminous aircraft and commercial exit signs Luminous dials, gauges, Liquid Crystal Displays (LCDs), and wrist watches Production of luminous paint Fuel for future fusion reactors In boosted fission, thermonuclear, enhanced neutron, and directed energy weapon devices Short half life implies the need to regularly remanufacture nuclear weapons
$_{92}$Uranium^{234}	$2.44x10^5$ y	In dental fixtures like crowns and dentures to provide a natural color and brightness
$_{92}$Uranium^{235}	$7.04x10^8$ y	Fuel for nuclear power plants and naval propulsion systems Early fission weapons devices Manufacture of fluorescent glassware Colored glazing for ceramics and wall tiles
$_{92}$Uranium^{238}	$4.468x10^9$	Predominant uranium isotope occurring with a 99.3 percent natural abundance Cannot be used to create a self sustained chain reaction Breeder material for breeding Pu^{239} in fission breeder reactors Shielding material against x-rays, neutron and gamma radiation Shielding armor against projectiles Kinetic energy projectiles in anti tank weapons Energy amplification in boosted fission and thermonuclear weapon devices
$_{54}$Xenon^{133}	5.25 d	In lung ventilation and blood flow studies in Nuclear Medicine

1.11 RADIOACTIVITY IN FOOD ITEMS

The accumulation in food of the isotopes of Ra^{226}, Th^{232}, K^{40}, C^{14} and T^3 causes a radiation equivalent dose to the human body averaging 20 mrem/year. The average banana fruit contains about 400 mg of potassium, leading to a specific activity of 3 pCi/gm from its K^{40} content. Brazil nuts are notorious for their radium content that causes a specific activity of 14 pCi/gm. Table 4 shows the specific activities in some food items.

Table 4. Specific activities and activity densities of some food items.

Food item	Specific Activity

Salad oil	4,900 pCi/l
Milk	1,400 pCi/l
Whiskey	1,200 pCi/l
Beer	390 pCi/l
Tap water	20 pCi/l
Brazil nuts	14.00 pCi/gm
Bananas	3.00 pCi/gm
Tea	0.40 pCi/gm
Flour	0.14 pCi/gm
Peanuts and peanut butter	0.12 pCi/gm

1.12 SUCCESSIVE RADIOACTIVE TRANSFORMATIONS

It can be observed that rarely does a radionuclide decay into other stable isotopes in a single step. Normally a chain of steps is encountered until a stable nuclide is reached. Consider the case of a radioactive isotope 1 decaying into another isotope 2, which in turn decays into a stable isotope 3. The rate equations for such a system are:

$$\begin{aligned} \frac{dN_1(t)}{dt} &= -\lambda_1 N_1(t) \\ \frac{dN_2(t)}{dt} &= +\lambda_1 N_1(t) - \lambda_2 N_2(t) \\ \frac{dN_3(t)}{dt} &= +\lambda_2 N_2(t) \end{aligned} \tag{17}$$

This is a coupled set of first order ordinary differential equations. The first equation has a simple solution obtained by separation of variables:

$$N_1(t) = N_{10}\, e^{-\lambda_1 t} \tag{18}$$

Inserting this equation into the second rate equation yields:

$$\frac{dN_2(t)}{dt} = +\lambda_1 N_{10}\, e^{-\lambda_1 t} - \lambda_2 N_2(t)$$

or:

$$\frac{dN_2(t)}{dt} + \lambda_2 N_2(t) = \lambda_1 N_{10}\, e^{-\lambda_1 t}$$

Multiplying by an integrating factor $e^{\lambda_2 t}$, we get:

$$e^{\lambda_2 t}\frac{dN_2(t)}{dt} + \lambda_2 N_2(t)e^{\lambda_2 t} = \lambda_1 N_{10} e^{-\lambda_1 t}e^{\lambda_2 t}$$

$$\frac{d[N_2(t)e^{\lambda_2 t}]}{dt} = \lambda_1 N_{10} e^{(\lambda_2-\lambda_1)t}$$

Separating the variables and integrating, we get:

$$\int_{N_{20}}^{N_2(t)} d[e^{\lambda_2 t}N_2(t)] = +\lambda_1 N_{10}\int_0^t e^{(\lambda_2-\lambda_1)t}dt$$

$$e^{\lambda_2 t} N_2(t) - N_{20} = + \frac{\lambda_1}{(\lambda_2-\lambda_1)} N_{10}[e^{(\lambda_2-\lambda_1)t} - 1]$$

Multiplying both sides by $e^{-\lambda_2 t}$,

$$N_2(t) = N_{20}e^{-\lambda_2 t} + \frac{\lambda_1}{(\lambda_2-\lambda_1)} N_{10}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) \quad (19)$$

If the initial number of nuclei N_{20} is zero, the equation reduces to:

$$N_2(t) = \frac{\lambda_1}{(\lambda_2-\lambda_1)} N_{10}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) \quad (20)$$

Upon substitution in the third rate equation, and assuming the initial number of nuclei N_{30} as zero, that the third member of the chain is stable, and integrating, we get:

$$N_3(t) = N_{10}[1 + \frac{\lambda_1}{(\lambda_2-\lambda_1)} e^{-\lambda_2 t} - \frac{\lambda_2}{(\lambda_2-\lambda_1)} e^{-\lambda_1 t})] \quad (21)$$

Assuming the half life of the first isotope is less than the half life of the second isotope, the overall result is that the number of nuclei of the isotope 1 will decrease exponentially according to its own half life. The second isotope number, which is initially zero, increases to a maximum and then decreases gradually. The third isotope as an end product will increase steadily with time and approaches N_{10}, since all the nuclei of the initial isotope will eventually decay to the stable end product.

1.13 SUCCESSIVE DECAYS GENERAL SOLUTION

As a generalization of the previous analysis, consider a radioactive decay chain containing n members.

$$\frac{dN_1(t)}{dt} = -\lambda_1 N_1(t)$$
$$\frac{dN_2(t)}{dt} = +\lambda_1 N_1(t) - \lambda_2 N_2(t)$$
$$\frac{dN_3(t)}{dt} = +\lambda_2 N_2(t) - \lambda_3 N_3(t) \qquad (22)$$
.....
$$\frac{dN_n(t)}{dt} = +\lambda_{n-1} N_{n-1}(t) - \lambda_n N_n(t)$$

Further assume that initially at time t=0:

$$N_{10} \neq 0$$
$$N_{20} = N_{30} = \ldots = N_{n0} = 0$$

This expresses a situation with a pure sample where only the parent substance ($N_{10} \neq 0$) is initially present.

We can write the product form of a general solution for this case as:

$$N_n(t) = \sum_{m=1}^{n} A_m e^{-\lambda_m t} \qquad (23)$$

where:

$$A_1 = \frac{\prod_{i=1}^{n-1} \lambda_i}{\prod_{i=2}^{n} (\lambda_i - \lambda_1)} . N_{10}$$

$$A_2 = \frac{\prod_{i=1}^{n-1} \lambda_i}{\prod_{\substack{i=1 \\ i \neq 2}}^{n} (\lambda_i - \lambda_2)} . N_{10}$$

……

$$A_m = \frac{\prod_{i=1}^{n-1} \lambda_i}{\prod_{\substack{i=1 \\ i \neq m}}^{n} (\lambda_i - \lambda_m)} . N_{10}$$

This form is a modified form of equations first derived by Bateman. The equations derived in the previous section can be directly derived from Eqn. 23, which lend themselves readily for numerical computations.

To obtain a more general solution for the case where:

$$N_{20}, N_{30} ... N_{n0} \neq 0,$$

the overall solution is obtained by adding to the solution above for $N_n(t)$ in an n-member chain, a solution for $N_n(t)$ in an (n-1)-member chain with now isotope 2 as the parent nuclide. Therefore $N_2(t) = N_{20}$ at t=0, and a general solution for $N_n(t)$ in an (n-2)-member chain, and the process is repeated.

When branching occurs in the radioactive decay chain, the decay constants in the numerators above should be replaced by the partial decay constants. Once the branches rejoin, the two branches are treated as separate chains. The two branches are followed, and then the contributions of the two paths are added out for any common member.

1.14 RADIOACTIVE EQUILIBRIA

SECULAR EQUILIBRIUM

Equilibrium is normally reached when the time derivatives in the rate equations are equal to zero, resulting in:

$$\begin{aligned}
&\frac{dN_1(t)}{dt} = -\lambda_1 N_1(t) = 0 \\
&\lambda_1 N_1(t) = \lambda_2 N_2(t) \\
&\lambda_2 N_2(t) = \lambda_3 N_3(t) \qquad (24) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\lambda_{n-1} N_{n-1}(t) = \lambda_n N_n(t)
\end{aligned}$$

This cannot be strictly achieved since this would lead to contradiction in the first equation when the decay constant would have to be equal to zero. However, a case close to equilibrium can be reached if the half life of the first parent is much longer than the half life of the daughter nuclide. This situation occurs in the naturally occurring decay chains. For instance, Uranium^{238} has a half life of 4.5 billion years. In this case we can take the number of atoms of the initial

parent N_1 as a constant, and the value of the decay constant is much smaller than other decay constants in the chain. This type of equilibrium is called "secular equilibrium," where:

$$\lambda_1 N_1(t) = \lambda_2 N_2(t) = \lambda_3 N_3(t) = \ldots = \lambda_{n-1} N_{n-1}(t) = \lambda_n N_n(t) \qquad (25)$$

This means that the activities of the chain members are equal:

$$A_1(t) = A_2(t) = A_3(t) = \ldots = A_{n-1}(t) = A_n(t) \qquad (26)$$

This situation applies whenever a succession of short lived isotopes arises from the decay of a relatively long lived parent.

This kind of equilibrium can also be attained when a radioactive substance is produced at a steady state from an artificial method, such as in a nuclear reactor or a particle accelerator.

The last expression can be used to determine the half life of an isotope having a long half life in terms of the half life of a short half life isotope, if we rewrite in the form:

$$\frac{N_1(t)}{T_1} = \frac{N_2(t)}{T_2} = \frac{N_3(t)}{T_3} = \ldots = \frac{N_{n-1}(t)}{T_{n-1}} = \frac{N_n(t)}{T_n} \qquad (27)$$

If the ratio of two isotopes in secular is known, and the half life of the daughter is known, then the half life of the parent can be calculated from:

$$T_1 = \left(\frac{N_1}{N_2}\right) T_2 \qquad (28)$$

EXAMPLE

The relative atomic abundance ratio for Uranium^{238} to Radium^{226} is 2.7925 million, and the half life for Radium^{226} is about 1,600 years, leading to:

$$\frac{N_{U^{238}}}{N_{Ra^{226}}} = 2.7925 \times 10^6$$

$$T_{Ra^{226}} = 1,600\ [years]$$

$$T_{U^{238}} = \left(\frac{N_{U^{238}}}{N_{Ra^{226}}}\right) T_{Ra^{226}}$$

$$= 2.7925 \times 10^6 \times 1,600\ [years]$$

$$= 4.468 \times 10^9\ [years]$$

We would like to study the approach to secular equilibrium. Consider a parent nuclide and its daughter, with the parent with a long half life such that:

$$T_1 >> T_2$$

or:

$$\lambda_1 << \lambda_2$$

Using the equation for the parent and daughter under this condition,

$$N_1 \cong N_{10}$$

$$N_2(t) \cong \left(\frac{\lambda_1}{\lambda_2}\right) N_{10} \left(1 - e^{-\lambda_2 t}\right)$$

This implies that the activity of the daughter reaches a saturation level according to the relationship:

$$A_2(t) = \lambda_2 N_2(t) \cong \lambda_1 N_{10} (1 - e^{-\lambda_2 t}) \qquad (29)$$

The last equation shows as the daughter's activity grows and eventually reaches the activity of the parent, as implied by secular equilibrium.

TRANSIENT EQUILIBRIUM

If the parent is long lived, but the half-life of the parent is not very long, the following condition is satisfied:

$$T_1 > T_2$$

or:

$$\lambda_1 < \lambda_2$$

Consequently:

$$N_2(t) \cong \left[\frac{\lambda_1}{(\lambda_2 - \lambda_1)}\right] N_{10}\, e^{-\lambda_1 t}$$

$$N_2(t) \cong \left[\frac{\lambda_1}{(\lambda_2 - \lambda_1)}\right] N_1(t)$$

$$\frac{N_2(t)}{N_1(t)} \cong \left[\frac{\lambda_1}{(\lambda_2 - \lambda_1)} \right] \tag{31}$$

In terms of the activities:

$$\frac{A_2(t)}{A_1(t)} \cong \frac{\lambda_2 N_2(t)}{\lambda_1 N_1(t)} \cong \frac{\lambda_2}{(\lambda_2 - \lambda_1)} \tag{32}$$

This implies that the daughter's activity is less than the parent's activity by the factor:

$$\frac{\lambda_2}{(\lambda_2 - \lambda_1)}.$$

CASE OF NO EQUILIBRIUM

If the parent is shorter lived than the daughter, the following condition is satisfied:

$T_1 < T_2$

or:

$\lambda_1 > \lambda_2$

and no state of equilibrium is attained. The daughter nuclide will increase, pass through a maximum, and decay with the half life of the daughter, as the parent decays with its own half life.

1.15 RADIOACTIVE DECAY CHAINS

Elements found in nature with an atomic number above Bismuth (Z=83), are radioactive. They belong to chains of successive transformations, starting from a radioactive element, and each ending with a stable isotope. There exist three series containing all the natural activities in this region of the Chart of the Nuclides. There is also an artificially created chain.

These chains are closely similar, and contain an interesting feature of branching transformations.

In each of these chains, we notice the occurrence of an isotope of radon. It is released in the form of a gas at room temperature. It constitutes a health hazard in uranium mines, in water wells using uranium containing aquifers, in natural gas, in homes built on rocks containing uranium, and in cigarette smoking.

THE URANIUM^{238} CHAIN

The parent substance in this case is U^{238}. It contains 14 transformations, 8 through alpha decay and 6 through beta emission. The end of the chain is the stable lead isotope Pb^{206}. This

chain contains radium and its decay products. The atomic mass number is here modified in units of four in each alpha transformation. The beta decays do not affect the mass number. A general formula for the mass number can be mathematically induced as:

$$(4n + 2)$$

where n is an integer. Radon^{222}, which is a health hazard in uranium mines and some human dwellings through its Po^{210} daughter, with a half life of 3.825 days occurs in this chain. This chain is shown in Fig. 3.

THE THORIUM^{232} CHAIN

The parent nuclide here is Th^{232}, and the stable end product is Pb^{208}. A general formula for the mass number can be mathematically induced as:

$$(4n)$$

where n is an integer.

Another radon isotope Rn^{220} with a half-life of 54.5 seconds appears in this chain. This chain is shown in Fig. 4.

THE URANIUM^{235}, ACTINIUM CHAIN

This chain has Pb^{207} as the stable end product and has U^{235} as its parent nuclide. A general formula for the mass number can be mathematically induced as:

$$(4n + 3)$$

where n is an integer. The radon isotope existing in this chain is Rn^{219} with a half-life of 3.92 seconds. This chain is shown in Fig. 5.

THE NEPTUNIUM^{237}, ARTIFICIAL CHAIN

This chain is artificially created and does not exist in nature. It starts with Np^{237} and ends with $_{83}$Bi^{209} as a stable element. Its general formula is:

$$(4n + 1).$$

This chain is shown in Fig. 6.

1.16 OTHER NATURALLY OCCURRING ISOTOPES

Other than the members of these chains, many radioactive isotopes have been discovered with long half-lives and small abundances, as shown in Table 4. Notably is K^{40}, with a half life

of 1.27 x 10^9 years, in the same range as that for U^{238} at 4.52 x 10^9 years. This isotope of potassium exists in living organic matter and cannot be separated from its other isotopes by chemical means (Table 5). The detection of these isotopes is rather difficult because of the existing radiation background in laboratories. This radiation background is caused by traces of uranium, thorium, potassium, and in a larger part due to cosmic radiation.

Table 5. Other naturally occurring or otherwise available radioisotopes.

Radioisotope	Isotopic Abundance (a/o)	Half-life (years)	Transformation type	Stable Products
T^3	-	12.33	$beta^-$	He^3
C^{14}	-	5370	$beta^-$	N^{14}
K^{40}	0.0117	1.27 x 10^9	$beta^-$,$beta^+$, ε	Ca^{40},Ar^{40}
V^{50}	0.25	1.40 x 10^{17}	$beta^-$, ε	Cr^{50},Ti^{50}
Rb^{87}	27.83	4.88 x 10^{10}	$beta^-$	Sr^{87}
Cd^{113}	12.22	7.7x10^{15}	$beta^-$	In^{113}
In^{115}	95.71	4.4 x 10^{14}	$beta^-$	Sn^{115}
Te^{123}	0.89	6.0 x 10^{14}	ε	Sb^{123}
La^{138}	0.09	1.05 x 10^{11}	ε, $beta^-$	Ba^{138},Ce^{138}
Ce^{142}	11.08	>5.0 x 10^{16}	alpha	Ba^{138}
Nd^{144}	23.8	2.38 x 10^{15}	alpha	Ce^{140}
Nd^{145}	8.3	>1.0 x 10^{17}	alpha	Pr^{141}
Sm^{147}	14.99	1.06 x 10^{11}	alpha	Nd^{143}
Sm^{148}	11.24	7.00 x 10^{15}	alpha	Nd^{144}
Sm^{149}	13.9	>1.00 x 10^{16}	alpha	Nd^{145}
Gd^{152}	0.20	1.1 x 10^{14}	alpha	Sm^{148}
Dy^{156}	0.057	>1.00 x 10^{18}	-	-
Lu^{176}	2.59	3.75x10^{10}	$beta^-$	Hf^{176}
Hf^{174}	0.16	2.0x10^{15}	alpha	Yb^{170}
Ta^{180m}	0.012	>1.2x10^{15}	ε, $beta^+$	Hf^{180},W^{180}
Re^{187}	62.6	4.12x10^{10}	$beta^-$	Os^{187}
Os^{186}	1.59	2.0x10^{15}	alpha	W^{182}
Pt^{190}	0.014	6.5x10^{11}	alpha	Os^{186}
Pb^{204}	1.42	1.4x10^{17}	alpha	Hg^{200}
Bi^{209}	100	>2.0x10^{18}	alpha	Tl^{205}
Th^{232}	100	1.40x10^{10}	alpha	Pb^{208}
U^{234}	0.0054	2.44x10^5	alpha	Pb^{206}
U^{235}	0.72	7.04x10^8	alpha	Pb^{207}
U^{238}	99.2745	4.468x10^9	alpha	Pb^{206}

ε = electron capture

1.17 SPONTANEOUS FISSION

Some heavy nuclei decay in a process where the nucleus breaks up into two intermediate mass fragments and several neutrons. It occurs in with nuclei with mass number A > 230.

Since the maximum binding energy per nucleon occurs at A = 60, nuclides above A > 100 are unstable with respect to spontaneous fission, sine a condition for spontaneous fission is:

$$m(A,Z) > m(A',Z') + m(A-A',Z-Z') \tag{33}$$

in the spontaneous fission reaction:

$${}_{Z}X^{A} \rightarrow {}_{Z'}X^{A'} + {}_{Z-Z'}X^{A-A'} \tag{34}$$

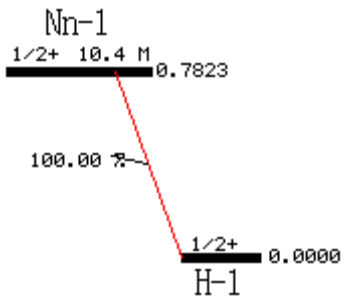
Because of the high Coulomb barrier for the emission of the fission fragments, spontaneous fission is only observed in the heaviest nuclei.

1.18 NEGATIVE BETA DECAY

Below the atomic number Z = 83, radioactive nuclides seek stability by either increasing decreasing their nuclear charge through either negative beta positron decay, or electron capture.

Nuclides possessing an excess number of neutrons, or neutron rich nuclides tend to undergo negative beta decay. Internally in this process, a neutron is transformed into a proton and a negative electron with the emission of an antineutrino for conservation of parity:

$${}_{0}n^{1} \rightarrow {}_{1}H^{1} + {}_{-1}e^{0} + \nu^{*} \tag{35}$$



The atomic number of the decaying nucleus is increased by one unit in this process:

$${}_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0} + \nu^{*} \tag{36}$$

For this process to occur, the condition for a negative beta decay to occur is:

$$m(A,Z) \geq m(A,Z+1) \tag{37}$$

The beta decay energy is defined as:

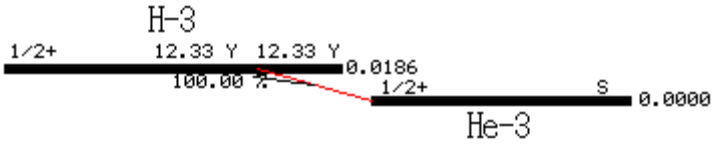
$$E(\beta^-) = m(A,Z) - m(A,Z+1)$$

It should be positive for the process of beta decay to be energetically possible. For instance, for Po^{210}, $E(\beta^-) = -3.981$ MeV, and in fact it decays through alpha decay into Pb^{206}.

Fission products, being neutron rich nuclei, undergo a succession of negative beta decays forming decay chains.

An example of a beta decay is the decay of the hydrogen isotope tritium:

$$_1T^3 \rightarrow {}_2He^3 + {}_{-1}e^0 + \nu^*$$



The beta decay energy for this reaction is 18.591 keV, which makes it energetically possible.

1.19 POSITRON DECAY

Nuclides possessing an excess number of protons, or proton rich nuclides tend to undergo a positron decay. Internally in this process, a proton is transformed into a neutron and a positron with the emission of a neutrino for conservation of parity:

$$_1H^1 \rightarrow {}_0n^1 + {}_{+1}e^0 + \nu \tag{38}$$

The atomic number of the decaying nucleus is decreased by one unit in this process:

$$_Z X^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + \nu \tag{39}$$

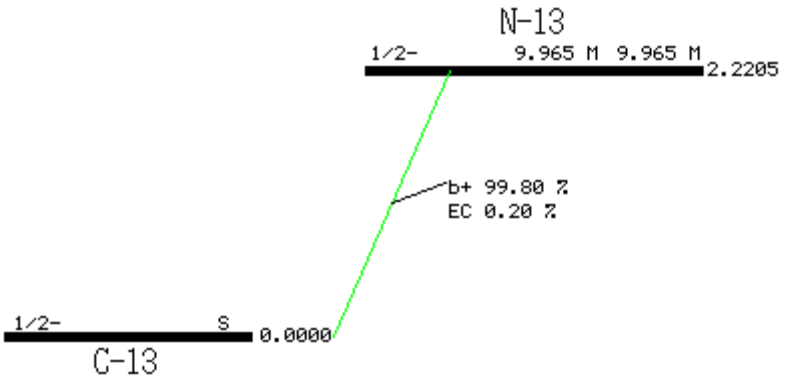
For this process to occur, the condition for a positron decay to occur is:

$$\begin{aligned} &m(A,Z) \geq m(A,Z+1) + 2m_e, \\ &m_e \text{ is an electron mass} = 0.51\ MeV. \end{aligned} \tag{40}$$

Fission products, being neutron rich nuclei, undergo a succession of negative beta decays forming decay chains.

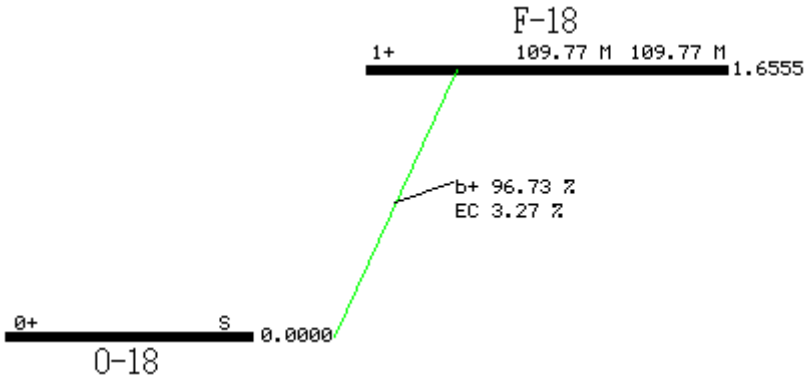
An example of a positron decay is the decay of the N^{13} isotope:

$$_7N^{13} \rightarrow {}_6C^{13} + {}_{+1}e^0 + \nu$$



Another example is the F^{18} positron decay used in Positron Emission Tomography or PET nuclear medicine scanning:

$$ {}_{9}F^{18} \rightarrow {}_{8}O^{18} + {}_{+1}e^{0} + \nu $$



1.20 ELECTRON CAPTURE

Similar to positron decay possessing an excess number of protons, or proton rich nuclides undergo an electron capture process. Internally in this process, a proton combines with an inner shell electron into a neutron with the emission of a neutrino for conservation of parity:

$$ {}_{1}H^{1} + {}_{-1}e^{0} \rightarrow {}_{0}n^{1} + \nu \tag{41} $$

The atomic number of the decaying nucleus is decreased by one unit in this process:

$$ {}_{Z}X^{A} + {}_{-1}e^{0} \rightarrow {}_{Z-1}Y^{A} + \nu \tag{42} $$

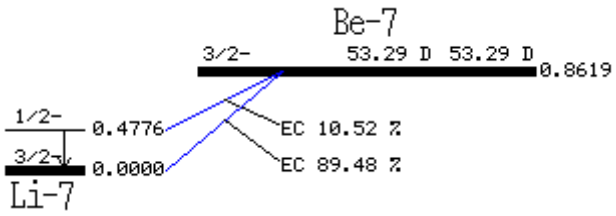
For this process to occur, the condition for a positron decay to occur is:

$$ m(A,Z) \geq m(A,Z-1) \tag{43} $$

Fission products, being neutron rich nuclei, undergo a succession of negative beta decays forming decay chains.

An example of an electron capture decay is the decay of the Be^7 isotope:

$$ {}_4Be^7 + {}_{-1}e^0 \rightarrow {}_3Li^7 + \nu $$



Positron decay is possible only if the initial and final masses differ by two electron masses = 2 m_e = 2 x 0.51 = 1.02 MeV. Electron capture, on the other hand, is possible if the initial mass is just larger than the final mass. If positron decay is possible, so is electron capture, and nuclei that cannot undergo positron decay can undergo electron-capture decay.

1.21 DOUBLE BETA DECAY

This is a rare radioactive event observed for Mo^{92} and Mo^{100}. Two beta particles and two antineutrinos are emitted, resulting in the original nucleus gaining two protons and losing two neutrons.

Nuclides such as Se^{82}, Cd^{116} and Te^{130} undergo this type of decay, albeit with half lives exceeding 10^{19} years

The decay process with the emission of two beta particles and no antineutrinos is possible according to theories requiring antineutrinos to have zero mass. Such decays have not been experimentally verified yet.

1.22 ALPHA DECAY

For nuclides with a large mass number A, alpha decay becomes possible.

$$ {}_Z X^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 \qquad (44 $$

For this process to occur, the condition for an alpha decay to occur is:

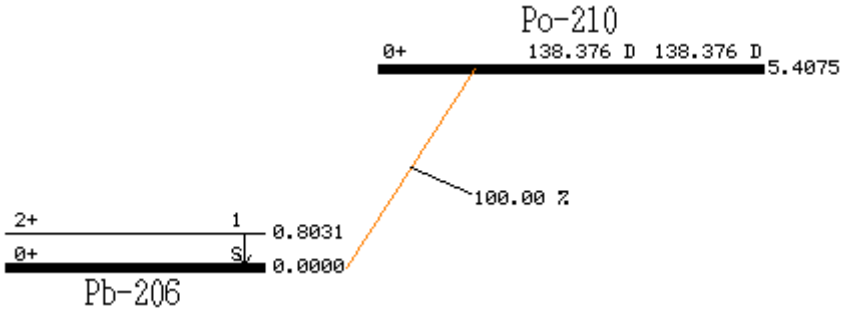
$$ m(A,Z) \geq m(A-4,Z-2) + m({}_2He^4) \qquad (45) $$

An example of an alpha decay is the decay of the Pu^{239} isotope:

$$ {}_{94}Pu^{239} \rightarrow {}_{92}U^{235} + {}_2He^4 $$

Another example is the decay of Po^{210} at the end of the U^{238} decay chain into the Pb^{206} stable lead isotope:

$$_{84}Po^{210} \rightarrow {}_{82}Pb^{206} + {}_{2}He^{4}$$

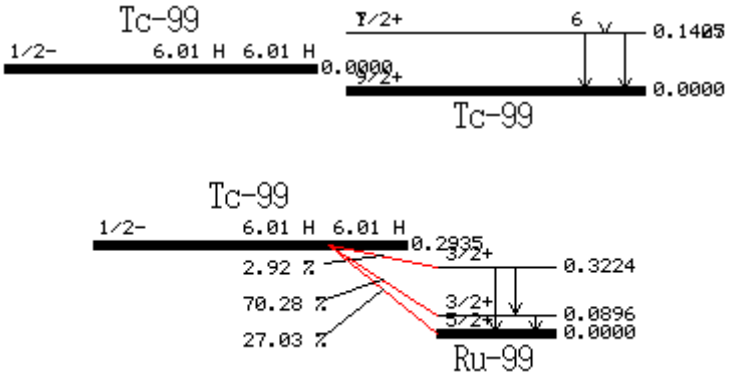


The binding energy per nucleon for the alpha particle is 7.1 MeV, consequently the total binding energy is 4 x 7.1 = 28.4 MeV. For some nuclides around A = 140, the biding energy per nucleon is around 7 MeV, and alpha decay is possible. It becomes a dominant decay mode for proton rich nuclides with A > 160 and for neutron rich nuclides with A > 180.

1.23 ISOMERIC TRANSITIONS

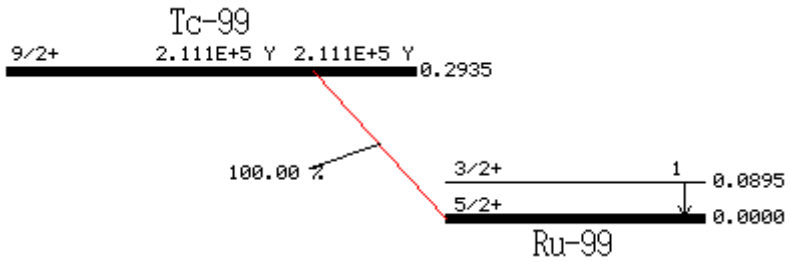
A metastable or isomeric state of a nuclide will decay to its ground state by an isomeric transition (IT) gamma ray emission followed by one or more gamma rays in a cascade. It is possible to have just one IT gamma ray to the ground state, or a more complicated scheme with more than one IT gamma ray, each with its cascading gamma rays. An example of an isomeric transition is the Technetium^{99m} decay with a half life of 6.02 hours:

$$_{43}Tc^{99m} \rightarrow {}_{43}Tc^{99} + \gamma$$



This is different from the normal Tc^{99} beta decay with a half life of 0.211×10^{6} years:

$$_{43}Tc^{99} \rightarrow {}_{44}Ru^{99} + {}_{-1}e^{0} + \nu^{*}$$



1.24 INTERNAL CONVERSION

This is a process in which the nucleus interacts with its extranuclear electrons. It competes with gamma ray emission. The excitation energy of the nucleus is usually transferred to a K shell orbital electron, and the electron is emitted from the atom instead of a gamma ray. Neither the atomic number Z nor the mass number A change in this process. The conversion electrons will have a kinetic energy equal to the difference between the energy of the nuclear transition involved and the binding energy of the electron in the atom. An example showing the competition with gamma rays emission occurs to Tc^{99m} as:

$$_{43}Tc^{99m} \rightarrow {}_{43}Tc^{99} + {}_{-1}e^{0}$$

1.25 DELAYED PARTICLE EMISSION

Nitrogen^{17} with a half life of 4.174 seconds decays through negative beta decay into short lived states of O^{17} which in turn emit neutrons. Thus N^{17} is considered to emit delayed neutrons with a half life of 4.174 seconds.

Delayed protons emission occurs in Si^{25} which decays by positron emission to its daughter Al^{25} which emits protons.

Delayed neutron emissions occur in some fission products, and greatly influence the control of fission reactors.

1.26 CLUSTER DECAY

Cluster decay has been observed in several heavy nuclides where clusters of C^{12}, C^{14}, O^{20}, Ne^{20}, Mg^{28}, or Si^{32} have been observed.

Table 6. Nuclides undergoing Cluster Decay.

Nuclide	Q value [MeV]	Cluster
Ba^{114}	18.3 – 20.5	C^{12}
Fr^{221}	31.28	C^{14}
Ra^{221}	32.39	C^{14}
Ra^{222}	33.05	C^{14}

Ra^{223}	31.85	C^{14}
Ra^{224}	30.54	C^{14}
Ac^{225}	30.48	C^{14}
Ra^{226}	28.21	C^{14}
Th^{228}	44.72	O^{20}
Pa^{231}	51.84	F^{23}
Th^{230}	57.78	Ne^{24}
Th^{232}	55.62, 55.97	Ne^{24}, Ne^{26}
Pa^{231}	60.42	Ne^{24}
U^{232}	62.31	Ne^{24}
U^{233}	60.50, 60.75	Ne^{24}, Ne^{25}
U^{234}	58.84, 59.47	Ne^{24}, Ne^{26}
U^{235}	57.36, 57.83	Ne^{24}, Ne^{25}
U^{236}	55.96, 56.75	Ne^{24}, Ne^{26}
U^{232}	74.32	Mg^{28}
U^{233}	74.24	Mg^{28}
U^{234}	74.13	Mg^{28}
U^{235}	72.20, 72.61	Mg^{28}, Mg^{29}
U^{236}	71.69, 72.51	Mg^{28}, Mg^{30}
Np^{237}	75.02	Mg^{30}
Pu^{236}	79.67	Mg^{28}
Pu^{238}	75.93, 77.03	Mg^{28}, Mg^{30}
Pu^{238}	91.21	Si^{32}
Pu^{240}	90.95	Si^{34}
Am^{241}	93.84	Si^{34}

The process starts with the formation of a cluster of nucleons within the nucleus followed by the cluster tunneling through the Coulomb barrier of the nucleus.

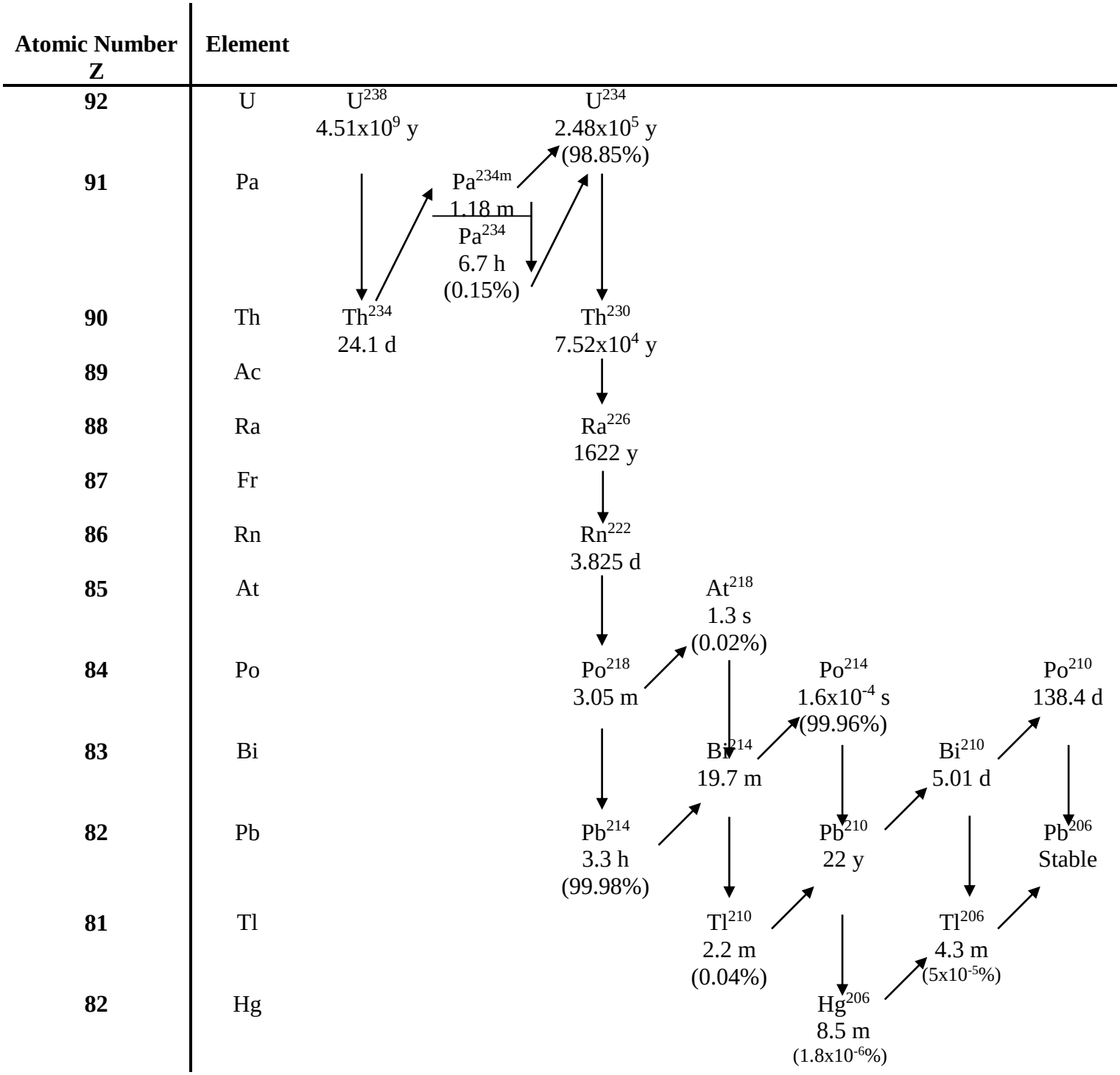


Figure 3. The uranium^{238} (4n + 2) radioactive decay series.

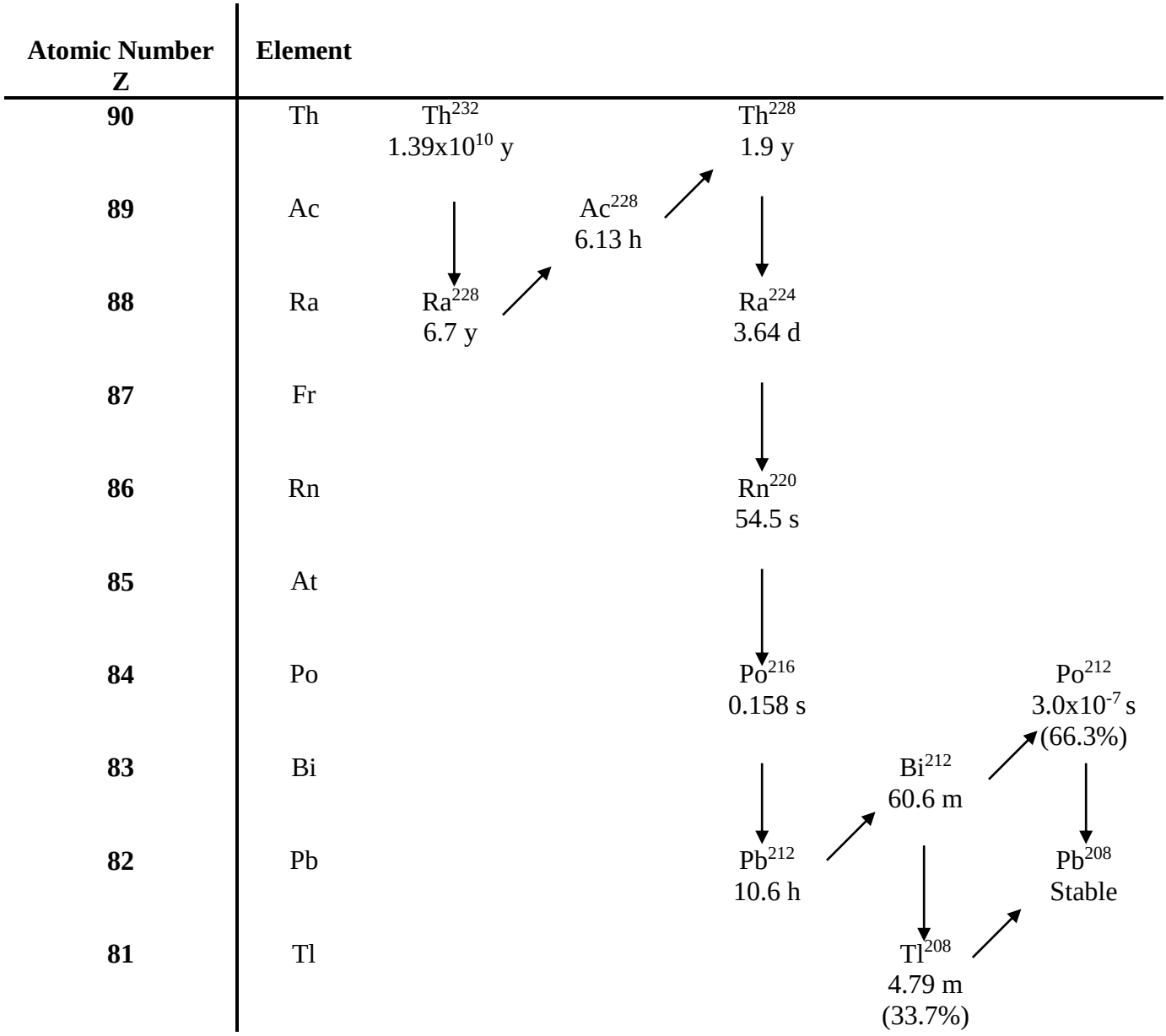


Figure 4. The thorium^{232} (4n) radioactive decay series.

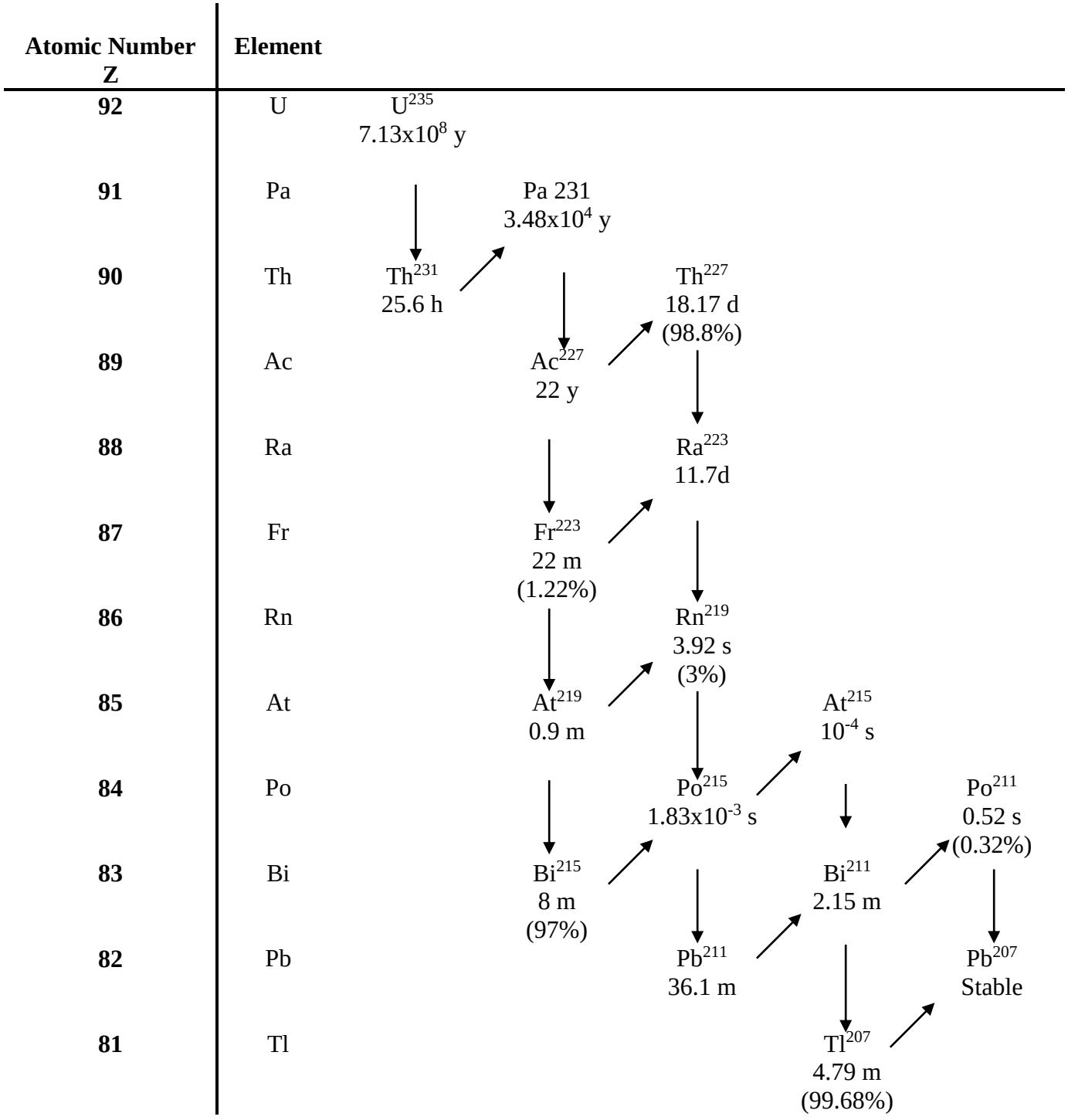


Figure 5. The uranium^{235} or actinium (4n + 3) radioactive decay series.

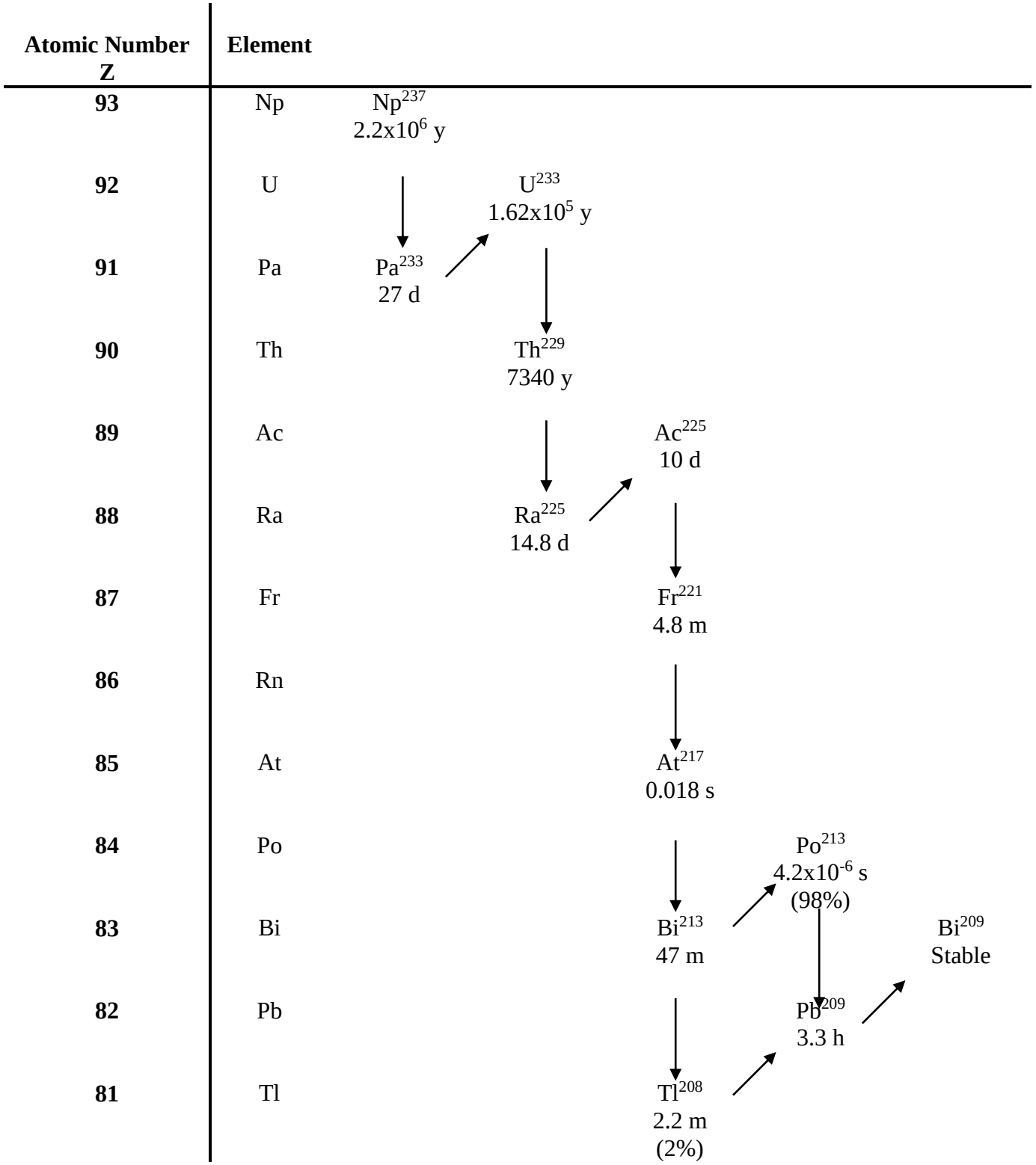


Figure 6. The artificial neptunium^{237} (4n + 1) radioactive decay series.

EXERCISES

1. Consider the isotope Ra^{226}. Using Avogadro's law, calculate its specific activity or the activity of 1 gram of material, and discuss its relationship to the Curie unit of activity. You can obtain the half life of the radium^{226} isotope from the Table of the Nuclides.

2. The naturally occurring isotope K^{40} is widely spread in the environment. In fact, the average concentration of potassium in the crustal rocks is 27 [g/kg] and in the oceans is 380 [mg/liter]. K^{40} occurs in plants and animals, has a half-life of 1.3 billion years and an abundance of 0.0119 atomic percent.

Potassium's concentration in humans is 1.7 [g/kg]. In urine, potassium's concentration is 1.5 [g/liter].

a) Calculate the specific activity of K^{40} in Becquerels per gram and in Curies/gm of K^{40}.

b) Calculate the specific activity of K^{40} in Becquerels per gram and in Curies per gm of overall potassium.

c) Calculate the specific activity of K^{40} in urine in [Bq/liter].

d) A beta activity above 200 transformations (disintegrations) per minute per liter of urine following accidental exposure to fission products is indicative of an internal deposition in the body, and requires intervention. How does this "body burden" criterion compare to the activity caused by the one due to the naturally occurring potassium?

3. The production of Carbon^{14} with a half life of 5730 years is an ongoing nuclear transformation from the neutrons originating from cosmic rays bombarding Nitrogen^{14} in the Earth's atmosphere:

$$
\begin{aligned}
{}_0n^1 + {}_7N^{14} &\rightarrow {}_1H^1 + {}_6C^{14} \\
{}_6C^{14} &\rightarrow {}_{-1}e^0 + {}_7N^{14} \\
\text{----------} & \\
{}_0n^1 &\rightarrow {}_{-1}e^0 + {}_1H^1
\end{aligned}
$$

where Nitrogen^{14} and Carbon^{14} appear as catalysts in the overall reaction leading to the disintegration of a neutron into a proton and an electron.

The atmospheric radiocarbon exists as $C^{14}O_2$ and is inhaled by all fauna and flora. Because only living plants continue to incorporate C^{14}, and stop incorporating it after death, it is possible to determine the age of organic archaeological artifacts by measuring the activity of the carbon present.

Two grams of carbon from a piece of wood found in an ancient temple are analyzed and found to have an activity of 20 disintegrations per minute. Estimate the approximate age of the wood, if it is assumed that the current equilibrium specific activity of C^{14} in carbon has been constant at 13.56 disintegrations per minute per gram.

4. Using the chart of the nuclides, generate the decay chains for U^{238} and Th^{232}.

a. Identify the two gaseous radon isotopes in the chain and find their decay graphs.

b. Identify the solid products of the radon chain that are of particular health interest, and show their decay diagrams, decay products, half lives, and decay energies.

5. Upon manufacturing for nuclear fuel, the decay chain of ${}_{92}U^{238}$ is broken during the process of the chemical reduction of Uranium Hexafluoride UF_6 into the uranium metal.

a. Calculate the specific activity of a fresh sample of uranium^{238}, which has a half life of 4.51×10^9 years, in Curies/gm and in Bq/gm.
b. The isotopes $_{90}Th^{234}$ and $_{91}Pa^{234m}$ are daughters of U^{238} and have as half-lives: 24.1 days and 1.18 minutes respectively. These half-lives are short compared with the half-life of the parent U^{238}. Hence they can be considered to be in "secular equilibrium" with the parent U^{238}. Calculate the total activity of a freshly manufactured sample of 1 gram of U^{238} under secular equilibrium conditions containing its daughters $_{90}Th^{234}$ and $_{91}Pa^{234m}$ in Curies and in Becquerels.

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