POSITION DEPENDENT AND ANGULAR SOURCE BIASING

© M. Ragheb
3/17/2013

1. INTRODUCTION

In particle transport simulations, situations arise when the source of particles has a geometry such that only a small fraction of the particles reaches the regions of interest in a given simulation. In this case the source particles need to be encouraged or biased into reaching and contributing to the answer at the detectors in the region of interest. Biasing is evidently a form of importance sampling.

This can be achieved assigning to the simulated an appropriate statistical weight that can be adjusted to compensate for the importance sampling or biasing process. The expected value of the statistical weight in the biased and the unbiased cases must be set to be equal in both cases.

Ragheb and Maynard [1-3] in a Monte Carlo particle transport shielding analysis of a tandem mirror conceptual fusion reactor design introduced the method of position dependent source biasing. Since it is an extension of the angular source biasing method with no position biasing, we shall consider here both methodologies.

2. ESTIMATION OF THE SOURCE STRENGTH

Monte Carlo calculations use a normalized source. Thus the estimated collision rate density is estimated per single source particle. To obtain the actual collision rate density, the Monte Carlo estimator must be renormalized at the end of the computation to the actual source strength.

Consider a fission or fusion nuclear reactor of power P in [MWth] with energy release per event of interest E in [MeV/event] and an associated number of source particles per event ν [particles/event]. The source strength S in [particles/sec] can be expressed as:

$$S = C\nu \frac{P}{E}.10^{6}\,\text{[source particles/sec]} \qquad (1)$$

where: $C = 0.62418\text{x}10^{13}$ is a conversion factor from Joules to MeV.

For instance if we consider a fusion reactor based on the DT fusion reaction:

$$_{1}D^{2} + {}_{1}T^{3} \rightarrow {}_{0}n^{1} + {}_{2}He^{4} + 17.62\,MeV \qquad (2)$$

In this case:

$$\nu = 1\ [neutron / fusion]$$
$$E = 17.6\ [MeV / fusion]$$

If the power of the reactor is:

$$P = 3,000\, MWth,$$

the according to Eqn. 1 the source strength is:

$$\begin{aligned} S &= 0.62418x10^{13}x1\frac{3,000}{17.62}10^{6} \\ &= 1.063x10^{21}[neutrons/\sec] \end{aligned}$$

If symmetry of the problem is taken into account, the source strength should further be modified by the value of the solid angle considered:

$$S' = S.\frac{\Delta\Omega}{4\pi} \tag{3}$$

For instance, if only one quadrant of the geometry is used to sample the source, then we can write:

$$\begin{aligned} S' &= S.\frac{2\pi/4}{4\pi} \\ &= \frac{S}{8} \end{aligned}$$

3. ANGULAR SOURCE BIASING

For an isotropic source the probability of a particle emerging through a solid angle Ω shown in Fig. 1 is given by:

$$p_1 = \frac{\Omega}{4\pi} \tag{4}$$

The probability of a particle emerging in the solid angle (4π-Ω) will in turn be:

$$\begin{aligned} p_2 &= 1 - p_1 \\ &= 1 - \frac{\Omega}{4\pi} \end{aligned} \tag{5}$$

Notice that according to Eqns. 4 and 5:

$$p_1 + p_2 = 1 \qquad (6)$$

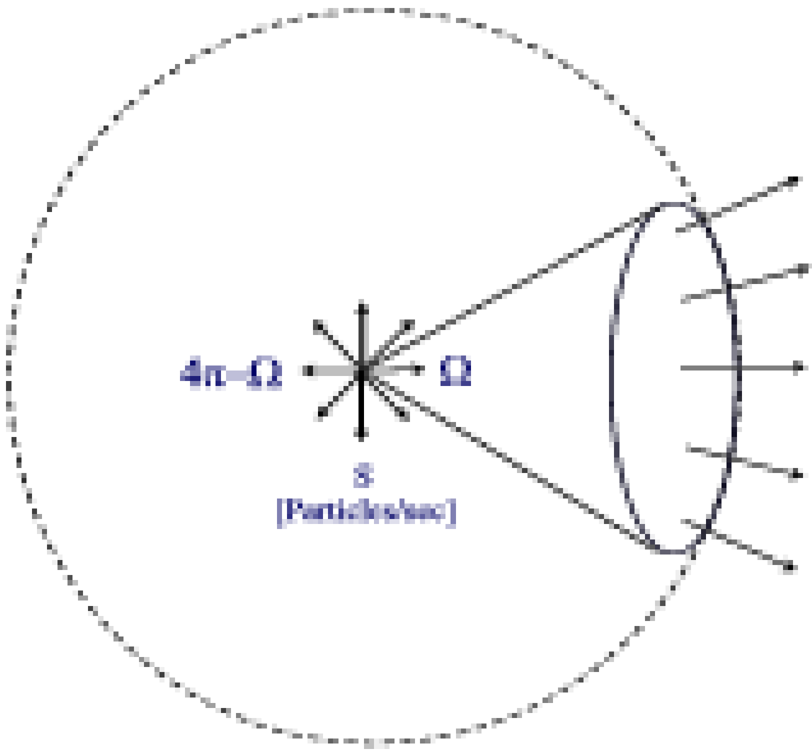

Figure 1. Biasing an isotropic source within the solid angle Ω.

One can assign arbitrary probabilities p'_1 and p'_2 for particle births in either the Ω or the (4π-Ω) solid angles, provided that their statistical weights w_1 and w_2 are adjusted so as to preserve the probabilities:

$$p_1' w_1 = p_1 \qquad (7)$$

and:

$$p_2' w_2 = p_2 \qquad (8)$$

such that:

$$p'_1 + p'_2 = 1 \qquad (9)$$

Substituting from Eqns. 4 and 5 into Eqns. 7 and 8 we get expression for the statistical weights to be assigned to the sampled biased particles:

$$w_1 = \frac{\Omega}{4\pi} \frac{1}{p'_1} \tag{10}$$

$$w_2 = \left(1 - \frac{\Omega}{4\pi}\right) \frac{1}{p'_2} \tag{11}$$

A special case of forward angular source biasing can be chosen with parameters:

$$\Omega = 2\pi$$
$$p'_1 = 1, \;\; p'_2 = 0.$$

In this case according to Eqns. 10 and 11:

$$w_1 = \frac{2\pi}{4\pi} . \frac{1}{1} = \frac{1}{2}$$

Thus the particles are only sampled in one hemisphere, however their statistical weight is adjusted from the unbiased value of unity to the biased value of ½ each.

If we choose instead to sample the particle 90 percent of the time in the forward direction and 10 percent in the backward direction, this would mean that:

$$p'_1 = 0.9, \;\; p'_2 = 0.1$$

and consequently their statistical weights are not unity anymore but in the forward direction instead:

$$w_1 = \frac{2\pi}{4\pi} . \frac{1}{0.9} = 0.5555$$

and in the backward direction:

$$w_2 = \frac{2\pi}{4\pi} . \frac{1}{0.1} = 5$$

Notice how the expected values of the probabilities p_1 and p_2 are preserved:

$$p_1 = p'_1 \, w_1 = 0.9x0.5555 = 0.5$$
$$p_2 = p'_2 \, w_2 = 0.1x5 = 0.5$$

4. POSITION DEPENDENT SOURCE BIASING

In the general case, the choice of the solid angle through which the source is sampled depends upon its position in the case of a distributed source. Consider for instance the cylindrical source of length L shown in Fig. 2. The detector is placed along the z-axis. Particles generated within the Ω solid angle are expected to reach the detector and contribute to the answer, whereas those generated within the (4π-Ω) solid angle may never reach the source. The approach involves the sampling of the axial position of the distributed source, determining the solid angle and the associated statistical weights, its radial position, its azimuthal position and its polar angle as shown below.

AXIAL POSITION SAMPLING

First, an axial position z is sampled uniformly along the cylinder from the probability density function:

$$p(z)dz = \frac{dz}{\int_0^L dz} = \frac{dz}{L} \tag{12}$$

The cumulative distribution function is:

$$C(z) = \frac{\int_0^z dz}{L} = \frac{z}{L} = \rho_1 \tag{13}$$

where: ρ_1 is a uniformly distributed random number over the unit interval.

The sampled axial position according to Eqn. 13 will be:

$$z = L.\rho_1 \tag{14}$$

SOLID ANGLES AND STATISTICAL WEIGHTS

Next, the solid angle subtended by the source upper end from the position z is calculated as:

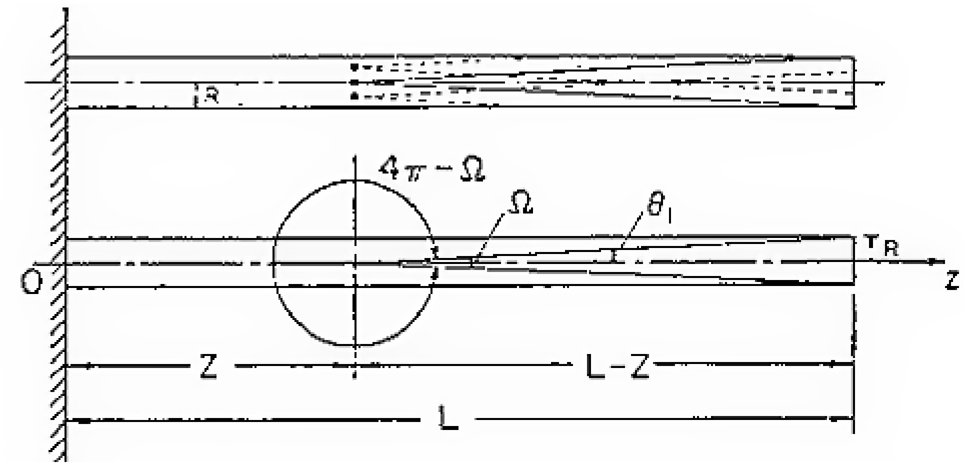

Figure 2. Sampling a cylindrical distributed source using position dependent source biasing.

$$
\begin{aligned}
\Omega &= \int d\Omega \\
&= \int \frac{dS}{r^2} \\
&= \int_0^{\theta_1} \int_0^{2\pi} \frac{r^2 \sin\theta}{r^2} d\vartheta d\phi \\
&= 2\pi(1 - \cos\theta_1) \\
&= 2\pi \left[1 - \frac{(L-z)}{\sqrt{(L-z)^2 + R^2}} \right] \\
&= 2\pi \left[1 - \frac{1}{\left(1 + \left\{ \frac{R}{(L-z)} \right\}^2 \right)^{1/2}} \right]
\end{aligned}
\tag{15}
$$

For the particle statistical weights we choose according to Eqns. 10 and 11:

$$
w_1 = \frac{\Omega}{4\pi} . \frac{1}{p'_1} = \frac{1}{2} \left\{ 1 - \frac{1}{\sqrt{1 + \left(\frac{R}{L-z} \right)^2}} \right\} . \frac{1}{p'_1} \tag{16}
$$

$$w_2 = \left(1 - \frac{\Omega}{4\pi}\right) \cdot \frac{1}{p'_2} = \left\{1 - \frac{1}{2}\left[1 - \frac{1}{\sqrt{1 + \left(\frac{R}{L-z}\right)^2}}\right]\right\} \cdot \frac{1}{p'_2} \qquad (17)$$

The choice of the sampling probabilities is left to the discretion of the user. For instance, Fig. 3 shows a choice of these probabilities of 90 percent in the forward direction and 10 percent in the backward direction.

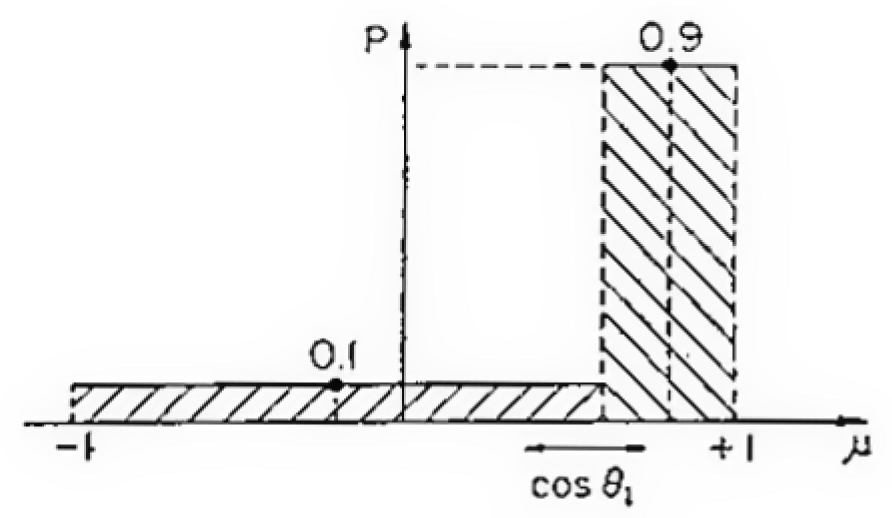

Figure 3. Choice of sampling probabilities in the primary and secondary directions.

SAMPLING THE RADIAL POSITION

The radial position of the source particle is sampled from the probability density function:

$$p(r)dr = \frac{2\pi r dr}{\int_0^R 2\pi r dr} = \frac{2}{R^2} r dr \qquad (18)$$

Its cumulative distribution function is:

$$C(r) = \frac{2}{R^2}\int_0^r r dr = \frac{2}{R^2} \cdot \frac{r^2}{2} = \frac{r^2}{R^2} = \rho_2 \qquad (19)$$

where: $\rho_2 \in [0,1]$.

Thus the sampled radial position of the source becomes:

$$r = R.\rho_2^{1/2} \tag{20}$$

SAMPLING THE AZIMUTHAL ANGLE

The azimuthal angle of the source can be sampled from the probability density function:

$$p(\phi)d\phi = \frac{d\phi}{\int_0^{2\pi} d\phi} = \frac{d\phi}{2\pi} \tag{21}$$

Its cumulative distribution function is:

$$C(\phi) = \frac{\int_0^{\phi} d\phi}{2\pi} = \frac{\phi}{2\pi} = \rho_3, \quad \rho_3 \in [0,1]. \tag{22}$$

Thus the sampled azimuthal angle becomes:

$$\phi = 2\pi.\rho_3 \tag{23}$$

SAMPLING THE POLAR ANGLE

The last step involves the sampling of the polar angle θ. This is sampled using the probability density function:

$$p_1(\theta)d\theta = \frac{2\pi \sin\theta d\theta}{\int_0^{\theta_1} 2\pi \sin\theta d\theta} = \frac{\sin\theta d\theta}{1 - \cos\theta_1} \tag{24}$$

which samples isotropic vectors within the conical half angle within the *primary* direction in the direction of the detector chosen for source sampling:

$$\theta_1 = \cos^{-1}\left[\frac{(L-z)}{\sqrt{R^2 + (L-z)^2}}\right] \tag{25}$$

For the secondary directions which does not go to the region of interest away from the detector:

$$p_2(\theta)d\theta = \frac{2\pi \sin\theta d\theta}{\int_{\theta_1}^{\pi_1} 2\pi \sin\theta d\theta} = \frac{\sin\theta d\theta}{\cos\theta_1 + 1} \tag{26}$$

The cumulative distribution function for the primary sampling direction is:

$$C_1(\theta) = \frac{\int_0^{\theta} \sin\theta d\theta}{1-\cos\theta_1} = \frac{1-\cos\theta}{1-\cos\theta_1} = \rho_4 \tag{27}$$

implying:

$$\theta = \cos^{-1}[1 + (1-\cos\theta_1)\rho_4] \tag{28}$$

The cumulative distribution function for the secondary sampling direction is:

$$C_2(\theta) = \frac{\int_{\theta_1}^{\theta} \sin\theta d\theta}{1-\cos\theta_1} = \frac{\cos\theta_1 - \cos\theta}{\cos\theta_1 + 1} = \rho_5 \tag{29}$$

implying:

$$\theta = \cos^{-1}[\cos\theta_1 - (1+\cos\theta_1)\rho_5] \tag{30}$$

5. DISCUSSION

Position dependent source biasing and ordinary source biasing are basically importance sampling methods. In both cases one can choose to sample a primary direction toward the region of interest or the detector with a large probability p'_1, while still obtaining the contribution from the regions of les interest by sampling them with a smaller probability p'_2.

The application of these methodologies is crucial for solving real world situations, which cannot be handled with direct sampling of the source. Any computation will be made more efficient by adopting them anyway, and consequently it is recommended that they be used whenever applicable. Care must be taken though to avoid under or overbiasing.

In particle transport biasing of the direction of travel is also applied to the transport kernel itself and not just to the source. This topic will deserve further attention.

EXERCISES

1. Calculate the source strength for the shielding calculation of a fission reactor of 3,000 MWth of power.
2. Prove that the expected values of the original probabilities are preserved in the case of position dependent source biasing in the same way that they are in the case of ordinary source biasing.

REFERENCES

1. M. Ragheb and C. W. Maynard, "Neutronics Shielding Analysis for the End Plugs of a Tandem Mirror Fusion Reactor," Journal of Fusion Energy, Vol. 1, No.4, 1981.
2. M. Ragheb, A. C. Klein and C. W. Maynard, "Neutronics Shielding Analysis of the Last Mirrors Beam Duct System for a Laser Fusion Power Reactor," Nucl. Technol. Fusion, Vol. 1:99, 1981.
3. M. Ragheb, "Optimized Fissile and Fusile Breeding in a laser fusion Fissile Enrichment Flux Trap Blanket," Journal of Fusion Energy, Vol. 1, No. 3, 1981.