TORQUE GENERATION IN WIND TURBINES

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INTRODUCTION

Wind turbines extract power from asymmetries in the wind stream by converting it into mechanical rotational energy of its rotors. This energy is transmitted to the loads such as a transmission or gearbox or an electrical generator through rotating shafts. These must be adequately designed to operate under normal and gusty wind conditions. They must be lubricated, cooled, properly aligned and free of vibrations.

SHAFT OPERATION

When a shaft is operated at an angular rotational speed ω, it is associated with the power P being transmitted through a torque T according to:

$$P = \omega . T \ [\frac{\text{radian}}{\text{sec}} . \frac{\text{Newton.meter}}{\text{radian}}],[\frac{\text{Joule}}{\text{sec}}],[\text{Watts}] \qquad (1)$$

This defines the torque as:

$$T = \frac{P}{\omega} \ [\frac{\text{Watt.sec}}{\text{radian}}],[\frac{\text{Joule}}{\text{radian}}],[\frac{\text{Newton.meter}}{\text{radian}}] \qquad (2)$$

This definition of the torque applies to a rotating shaft, and is different from the torque on a stationary structure such as a structural tower which would have units of Newton.meter.

STRESS GENERATION

As torque is applied to a rotating shaft, internal stresses or pressures are applied on the shaft material. Since this stress tends to shear instead of stretching or compressing the shaft, it is designated as a shear stress. The shear stress in a solid shaft is a function of the radial position r on the shaft axis and is largest at the shaft's surface. In terms of the polar moment of inertia J of the shaft it is given by:

$$\sigma_s(r) = \frac{T}{J} r [\frac{\text{Newton.meter.radian}}{\text{radian.m}^4}.\text{meter}],[\frac{\text{Newton}}{\text{m}^2}],\text{Pascal,Pa} \qquad (3)$$

where the polar moment of inertia of a solid shaft with radius r_0 is given by:

$$J = \frac{1}{2}\pi r_0^4 [\frac{\mathrm{m}^4}{\mathrm{radian}}] \tag{4}$$

MAXIMUM SHEARING STRESS

A rotating shaft must be designed so as to carry a given torque T by determining the maximum shearing stress that can be allowed for a chosen shaft material. The maximum stress occurs at the surface of the shaft when r = r_0. The shaft radius that will carry this maximum stress from Eqns. 3, 4, is given by:

$$\sigma_s(r_0) = \frac{T}{J} r_0 = \frac{T}{\frac{1}{2}\pi r_0^4} r_0 = \frac{2T}{\pi r_0^3}$$

$$r_0^3 = \frac{2T}{\pi \sigma_s(r_0)}$$

From which:

$$r_0 = \sqrt[3]{\frac{2T}{\pi \sigma_s(r_0)}} [\mathrm{meter}] \tag{5}$$

Significant safety and ignorance factors need to be introduced at that stage.

TRANSMISSION, GEARBOX DESIGN

Consider the design of a wind generator with an electrical output of:

$$P_e = 0.5 MWe$$

Accounting for the generator efficiency, the power at the transmission output would be:

$$P_t = \frac{P_e}{\eta_g} \tag{6}$$

For a generator efficiency of 90 percent, this would be:

$$P_t = \frac{500,000}{0.90} = 555,555\ Watts$$

And the power at the transmission input would be:

$$P_m = \frac{P_t}{\eta_t} = \frac{P_e}{\eta_g \eta_t} \qquad (7)$$

For a transmission efficiency of 90 percent, this would be:

$$P_m = \frac{500,000}{0.9x0.9} = 617,284\ Watts$$

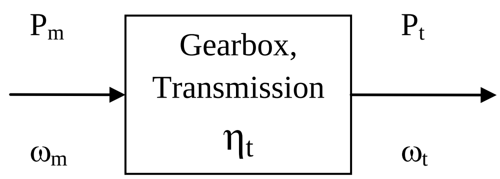

Fig. 1: Power and rotational speeds across a transmission or gear box.

Taking the rotational speed of the generator at 1,200 rpm, yields:

$$\omega_t = 2\pi \frac{1,200}{60} = 40\pi [\frac{\text{radians}}{\text{sec}}]$$

Taking the rotational speed of the rotor shaft as 24 rpm, corresponding to a gearing ratio of:

$$Gearing\ ratio : GR = \frac{1,200}{24} = 50$$

yields:

$$\omega_m = 2\pi \frac{24}{60} = \frac{4}{5}\pi [\frac{\text{radians}}{\text{sec}}]$$

From Eqn. 2 the torques at the high speed and low speed shafts torques become:

$$T_t = \frac{P_t}{\omega_t} = \frac{555,555}{40\pi} = 4{,}421[\frac{\text{N.m}}{\text{rad}}]$$

$$T_m = \frac{P_m}{\omega_m} = \frac{617,284}{\frac{4}{5}\pi} = 245{,}610\ [\frac{\text{N.m}}{\text{rad}}]$$

A maximum stress for steel shafts is recommended as 55 Mpa. Accounting for a factor of safety FS of 3 and an ignorance factor IF of 2 yields for the design maximum stress:

$$\sigma_{s,\max}(r_0) = \frac{\sigma_s(r_0)}{FS.IF} = \frac{\sigma_s(r_0)}{3x2} = \frac{\sigma_s(r_0)}{6}$$

$$\sigma_{s,\max}(r_0) = \frac{55}{6} = 9.2MPa$$

Substituting in Eqn. 5 yields for the high speed and low speed shaft radii:

$$r_{0,t} = \sqrt[3]{\frac{2T_t}{\pi\sigma_{s,\max}(r_0)}} = \sqrt[3]{\frac{2\times 4,421}{\pi\times 9.2x10^6}} = \left[305.9\times 10^{-6}\right]^{1/3} = 6.738\times 10^{-2}[\text{meter}] = 6.7\text{ cm}$$

$$r_{0,m} = \sqrt[3]{\frac{2T_m}{\pi\sigma_{s,\max}(r_0)}} = \sqrt[3]{\frac{2\times 245,610}{\pi\times 9.2x10^6}} = \left[16,995.8\times 10^{-6}\right]^{1/3} = 25.709\times 10^{-2}[\text{meter}] = 25.7\text{ cm}$$

Notice the difference in the radii of the high speed shaft (6.7 cm) and the low speed shaft (25.7 cm). Because of its size and weight, the low speed shaft must be kept at a minimum length in wind turbines designs.