SIMILARITY, SCALING AND MODEL TESTING

2/22/2009

INTRODUCTION

To test the forces acting on a given wind turbine design, there may be a need to test a model before building a prototype of it in a wind tunnel setting, much like airplanes models are tested before building a full scale version of the model as a prototype.

GEOMETRIC AND KINEMATIC SIMILARITY

For an adequate assessment of a model in a wind tunnel environment, it must possess a geometrical as well as a kinematic and dynamic similarity to the eventual full scale design.

The model would be under kinematic similarity if the incidence angles of the wind on each corresponding rotor blade element of the model and the prototype are the same.

SIMILITUDE LAWS

The laws of similitude can be obtained by expressing the geometrical similarity of the profiles, the kinematic similitude expressed as the same shapes for the speed triangles, and the dynamic similitude expressing the forces acting on the model and the prototype must be in the same ratio whatever their nature is. That includes viscosity, density, inertia and pressure.

In theory, the viscosity forces must be taken into account. This amounts to the equality of the Reynolds numbers for the model and the prototype:

$$\mathrm{Re}_V = \frac{V_1 D_1}{\mu_1} = \frac{V_2 D_2}{\mu_2} \tag{1}$$

where: 1 refers to the model,
2 refers to the prototype,
D_1, D_2 are the dimensions of the model and prototype respectively,
μ_1, μ_2 are the viscosities at the model and prototype respectively,
V_1, V_2 are the wind speeds through the rotors of the model and prototype.

In addition:

$$\text{Re}_U = \frac{U_1 D_1}{\mu_1} = \frac{U_2 D_2}{\mu_2}$$

where: U_1, U_2 are the circumferential speed of the corresponding rotors elements in model and prototype, respectively. (2)

It is impossible to consider this condition for the large industrial wind turbines. Since the model tests would be conducted in wind tunnels at atmospheric pressure and ambient temperature, it results that the viscosity:

$$\mu_1 = \mu_2 \tag{3}$$

And the preceding equations can be written as:

$$V_1 D_1 = V_2 D_2 \tag{4}$$

and:

$$U_1 D_1 = U_2 D_2 \tag{5}$$

Equation 4 suggests that the model should be tested in a wind tunnel with a wind speed of:

$$V_2 = V_1 \frac{D_1}{D_2} \tag{6}$$

which is higher than V_1.

In addition, the circumferential speed U can be expressed in terms of the rotational speed N as:

$$U = 2\pi NR = \pi ND \tag{7}$$

This allows us to write Eqn. 5 as:

$$\begin{aligned} \pi N_1 D_1 . D_1 &= \pi N_2 D_2 . D_2 \\ \pi N_1 D_1^{\,2} &= \pi N_2 D_2^{\,2} \end{aligned} \tag{8}$$

From which the rotational speed of the model must be:

$$N_2 = N_1 \frac{D_1^{\ 2}}{D_2^{\ 2}} \tag{9}$$

which is much higher than that of the prototype.

Using computational fluid dynamics simulations of the wind tunnel may help here since achieving these conditions in a conventional wind tunnel are rather difficult.

WIND TUNNEL CONSIDERATIONS

Suppose we have a scale model which 1/10 of the original wind turbine that has a diameter of 10 meters and a wind rotor tip speed ratio of 6 operating in a wind at a speed of 8.7 m/s and a rotational speed of 100 rpm.

In this case:

$$D_1 = 10\,m$$

$$\lambda = 8.7$$

$$\frac{D_2}{D_1} = \frac{1}{10},$$

$$\lambda = 6$$

$$N_1 = 100\,rpm$$

$$U_1 = 8.7\,m/s$$

$$D_2 = \frac{D_1}{10} = \frac{10}{10} = 1\,m$$

$$U_1 = 2\pi N_1 R_1 = \lambda V_1$$