

# EVENT TREE ANALYSIS

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## INTRODUCTION

Event trees are related to, but significantly different from decision trees. In general, decision trees are the representation of a process in which the adequacy of the tree depends principally on the skill and judgment of the safety analyst in properly conceptualizing the problem under consideration.

While this type of skill applies to the event trees analysis in reactor safety studies, the analyst is aided considerably because the elements of the trees are physical entities that exist in the nuclear power plant and the processes involved in the tree follow engineering and physical principles.

The understanding of the details of plant design and of the associated physical principles, aid the analyst greatly in ensuring a proper conceptualization for the reactor event trees.

## DECISION TREES

An example of a decision tree is shown in Fig. 1, and involves the following decision problem: An oil wildcatter must choose between drilling a well and selling his rights in a given location.

In a real world situation there would be many more acts, such as selling partial rights, sharing risks, farm-outs, etc. The desirability of drilling depends on the amount of oil which will be found.

For simplification, we consider the final binary state:

$$\theta : \{\text{Oil, No Oil}\} : \{\theta_1, \theta_2\} \quad (1)$$

Before making his decision, the oil wildcatter can, if he wishes, obtain more geological and geophysical evidence by means of expensive experiments. This is described by the subspace:

$$e : \{\text{Geological survey, No geological survey}\} : \{e_0, e_1\} \quad (2)$$

We simplify again by allowing for only one form of experiment: seismographic recordings. We also assume that these recordings will give completely reliable information that one of these conditions could prevail:

$z$  : {No geological survey results available,  
 There is no subsurface structure,  
 There is an open subsurface structure,  
 There is a closed subsurface structure} :  $\{z_0, z_1, z_2, z_3\}$ 
(3)

The wildcatter is next faced with the decision to drill or not to drill:

$a$  : {Drill, Do not drill} :  $\{a_1, a_2\}$ 
(4)

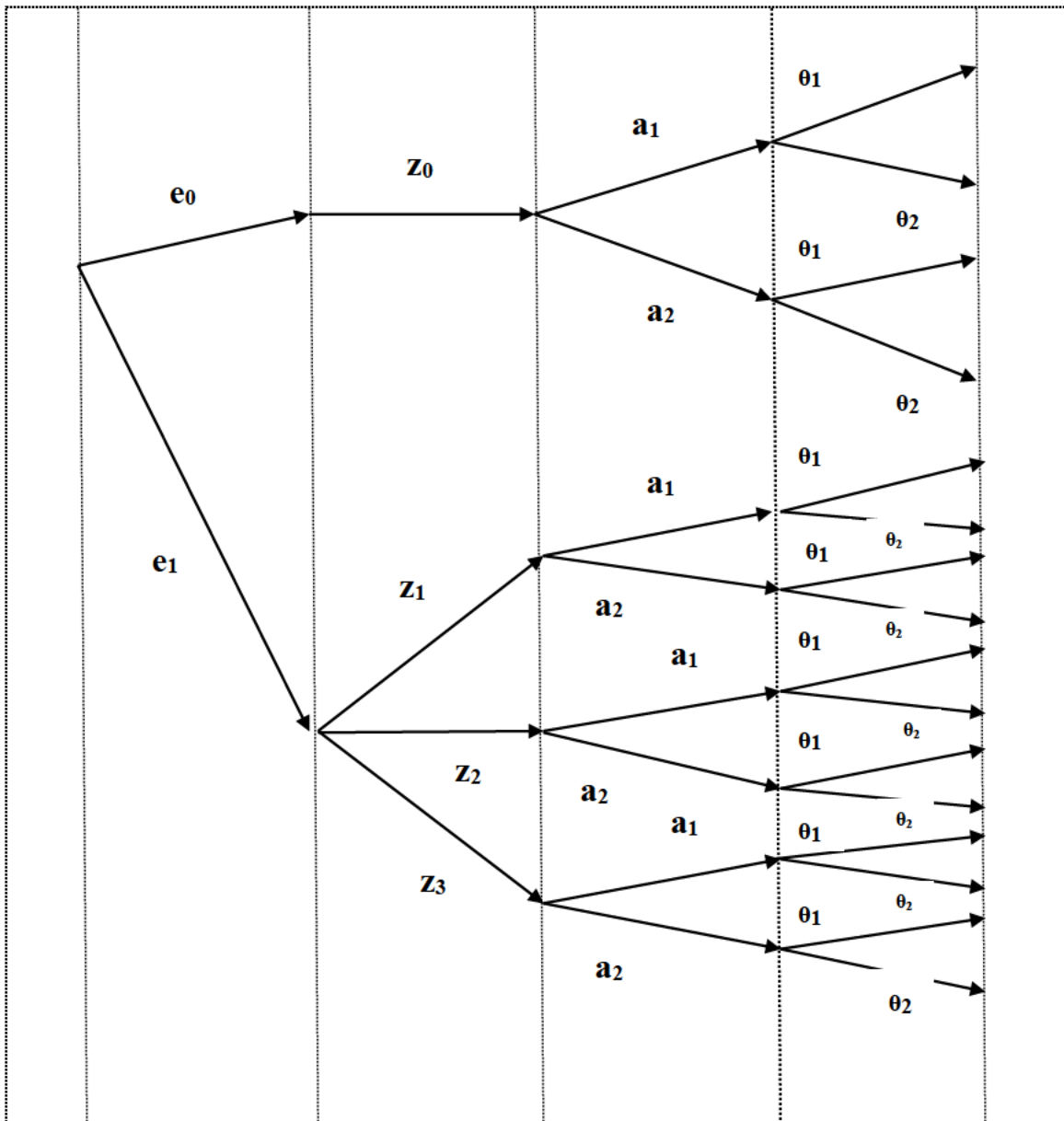


Figure 1. Decision Tree for the decision space:  $\{e, z, a, \theta\}$ .

The decision space can thus be represented as:

$$\{e, z, a, \theta\} \quad (5)$$

We notice that the three possible outcomes of the subsurface structure correspond to the states  $\{z\}$ , with the final outcome represented by the states  $\{\theta\}$ .

At each node in the decision tree “utility values” can be added, with the final path choice made according to the largest value of the utility function in the outcome space  $\{\theta\}$ .

## EVENT TREE ANALYSIS

The construction of an event tree involves several steps:

1. The identification of an initiating event for a postulated accident sequence and its probability or possibility of occurrence.
2. Determining the different components of the considered system affected by the initiating event.
3. Deriving the accidents sequences through the different system components assuming the two binary states of:

$$\{\text{Success state, Failure state}\} \quad (6)$$

4. To avoid a situation of binary explosion, prune the trees of the illogical accident sequences.
5. Assigning probabilities or possibilities for the success and failure states.
6. Deriving the Boolean expression for each accident sequence assuming an AND logical gate.
7. Calculating the probability or possibility of each accident sequence.

An event tree with an initiating event I, and two system components 1 and 2, with their associated accident sequences is shown in Fig. 2.

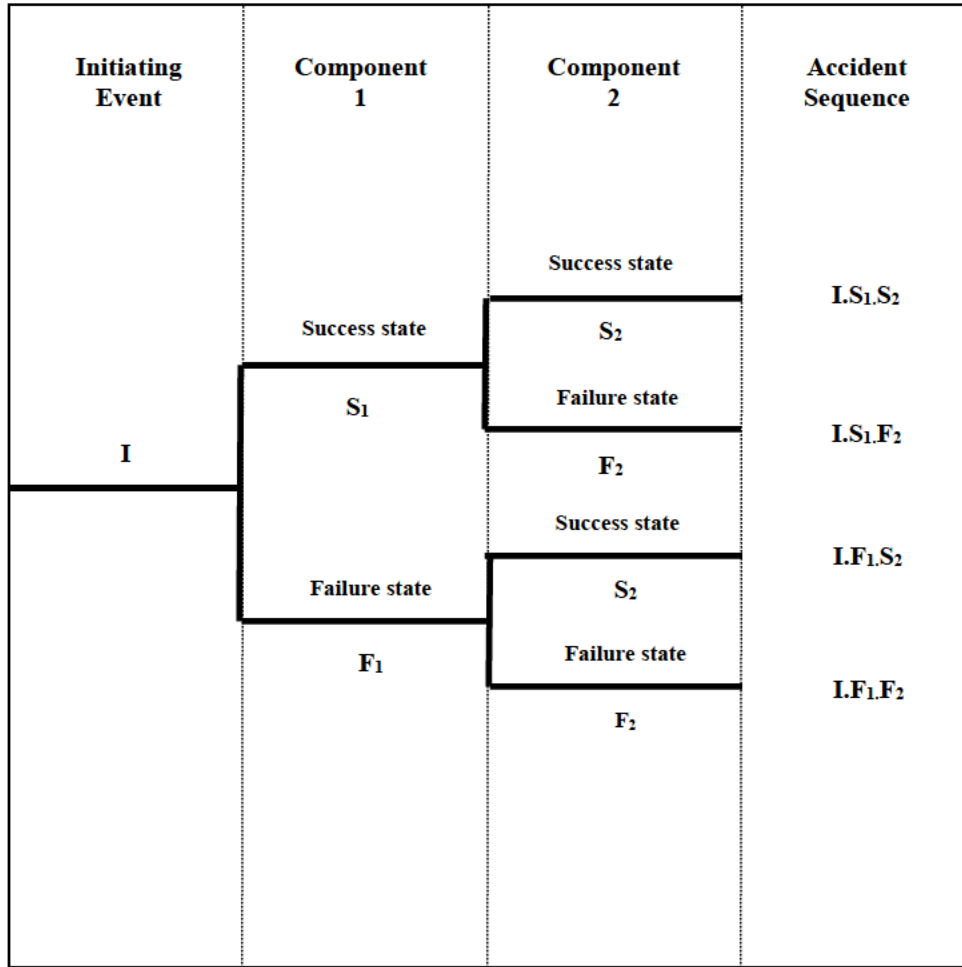


Figure 2. Event Tree for a two-component system showing the accident sequences.

### SEQUENCES FAILURE PROBABILITIES

Once the accident sequences are identified, the next step is to estimate the probability of failure of each sequence.

We first identify the binary nature of the system where the sum of the probabilities of failure and success add up to unity:

$$P(S) + P(F) = 1 \quad (7)$$

From which:

$$P(S) = 1 - P(F) \quad (8)$$

For each accident sequence, assuming independent events and an AND logical Boolean expression, the probability of occurrence of the accident sequence can be written as:

$$\begin{aligned}
 P(I.F_i.S_i) &= P(I.AND.F_i.AND.S_i) \\
 &= P(I)P(F_i)P(S_i)
 \end{aligned}
 \tag{9}$$

Substituting from Eqn. 8 into Eqn. 9 leads to:

$$P(I.F_i.S_i) = P(I)P(F_i)P(1 - F_i)
 \tag{10}$$

This leads to an event tree in terms of the probabilities of occurrences of the different accident sequences shown in Fig. 3.

Probability of Initiating Event	Probabilities for Component 1	Probabilities for Component 2	Accident sequences probabilities
<b>P(I)</b>	<b>1-P(F<sub>1</sub>)</b>	Success state	<b>P(I)[1-P(F<sub>1</sub>)] [1-P(F<sub>2</sub>)]</b>
		<b>1-P(F<sub>2</sub>)</b>	
	Failure state	<b>P(I)[1-P(F<sub>1</sub>)] P(F<sub>2</sub>)</b>	
	<b>P(F<sub>2</sub>)</b>		
	Success state	<b>P(I)P(F<sub>1</sub>) [1-P(F<sub>2</sub>)]</b>	
	<b>1-P(F<sub>2</sub>)</b>		
<b>P(F<sub>1</sub>)</b>	Failure state	<b>P(I)P(F<sub>1</sub>) P(F<sub>2</sub>)</b>	
	<b>P(F<sub>2</sub>)</b>		

Figure 3. Probabilities of the accident sequences assuming logical AND Boolean expressions.

### SEQUENCES FAILURE POSSIBILITIES

In case we are dealing with possibilities rather than probabilities, the possibility of theory expressions become:

$$\Pi(S) + \Pi(F) = 1 \quad (7)'$$

$$\Pi(S) = 1 - \Pi(F) \quad (8)'$$

$$\begin{aligned} \Pi(I.F_i.S_i) &= \Pi(I.AND.F_i.AND.S_i) \\ &= \text{Min}\{\Pi(I), \Pi(F_i), \Pi(S_i)\} \end{aligned} \quad (9)'$$

$$\Pi(I.F_i.S_i) = \text{Min}\{\Pi(I), \Pi(F_i), \Pi(1 - F_i)\} \quad (10)'$$

This results into the Event Tree shown in Fig. 4.

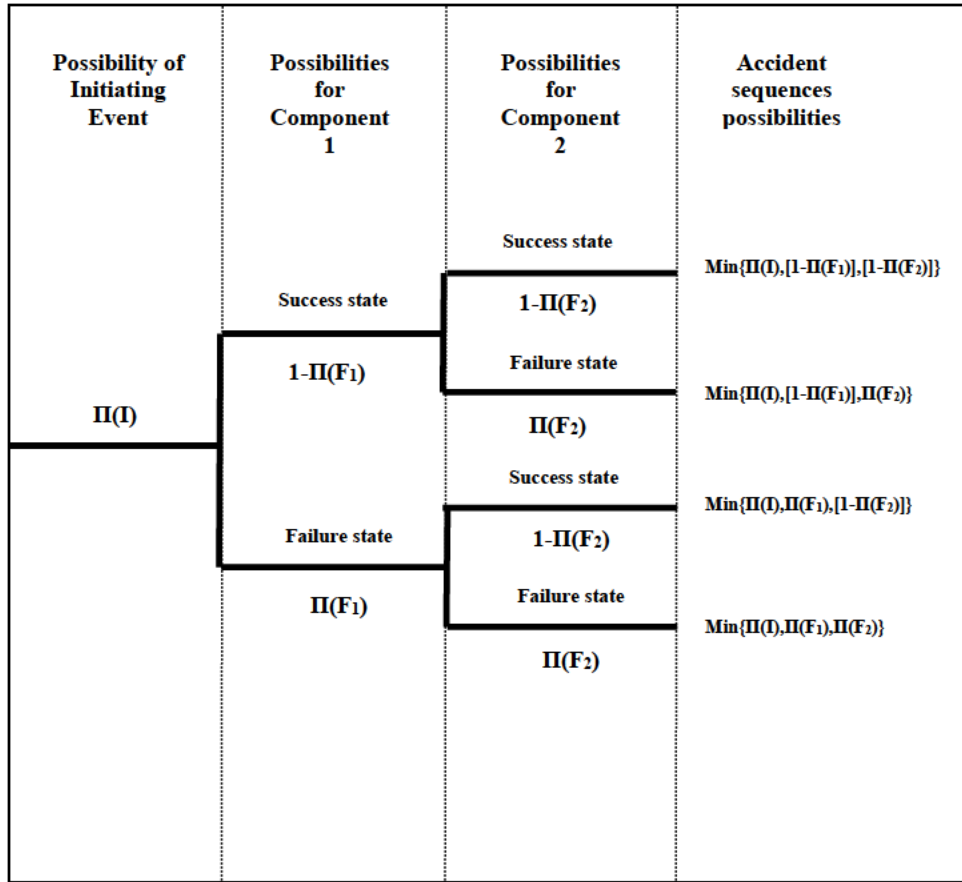


Figure 4. Possibilities of failure in a two component system.

### THE SMALL FAILURE PROBABILITIES AND POSSIBILITIES APPROXIMATION

It is necessary to use any available valid simplifications in the modeling of complex systems.

Whenever the probabilities of failures of the considered components are small, say in the range of  $10^{-4}$ , which is usually the case, Eqn. 8 can be written as:

$$P(S) = 1 - P(F) \approx 1, \forall P(F) \leq 10^{-4} \quad (8)''$$

This leads to a simplified event tree as shown in Fig. 5.

Probability of Initiating Event	Probabilities for Component 1	Probabilities for Component 2	Accident sequences probabilities
$P(I)$	$\approx 1$	Success state	$P(I)$
		$\approx 1$	$P(I)P(F_2)$
	$P(F_1)$	Success state	$P(I)P(F_1)$
		$\approx 1$	$P(I)P(F_1)P(F_2)$
	Failure state	$P(F_2)$	
	Failure state	$P(F_2)$	

Figure 5. Probabilities of the accident sequences assuming logical AND Boolean expressions using the small probabilities approximation.

Similarly, whenever the possibilities of failures of the considered components are small, say in the range of  $10^{-4}$ , Eqn. 8' can be written as:

$$\Pi(S) = 1 - \Pi(F) \approx 1, \forall \Pi(F) \leq 10^{-4} \quad (8)'''$$

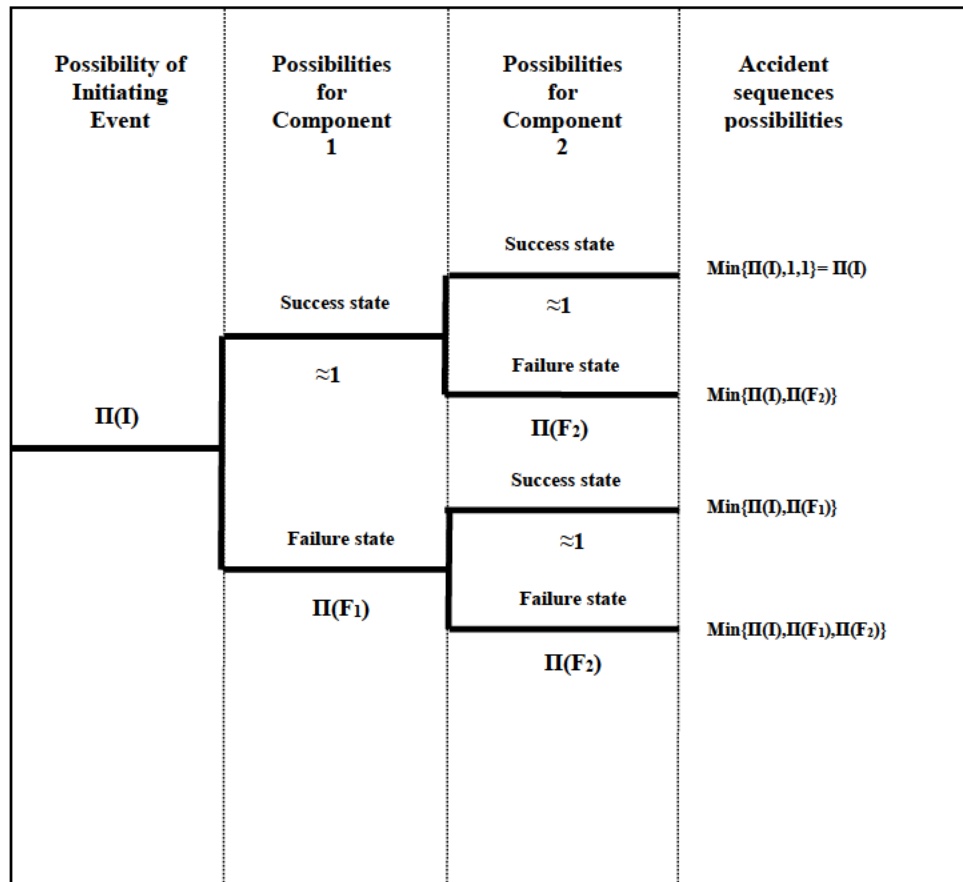


Figure 6. Possibilities of failure in a two component system using the small possibilities approximation

## BASIC EVENT TREE OF A LOSS OF COOLANT ACCIDENT, LOCA

We consider as an application of an Event Tree the situation of an initiating event with an associated failure probability:

Probability of initiating event A:  $P(I) = P(\text{Small pipe break})$

The probabilities of failure of the other components that would be part of the sequence of events leading to an eventual release of radioactivity are:

Probability of failure of component A:

$P(A) = P(\text{Electric Power Availability})$

Probability of failure of component B:

$P(B) = P(\text{ECCS}) = P(\text{Emergency Core Cooling System})$



Probability of failure of component C:  
 $P(C) = P(\text{Fission products removal})$

Probability of failure of component D:  
 $P(D) = P(\text{Containment leakage})$

The corresponding Event Tree is shown in Fig. 7 using the small probabilities approximation.

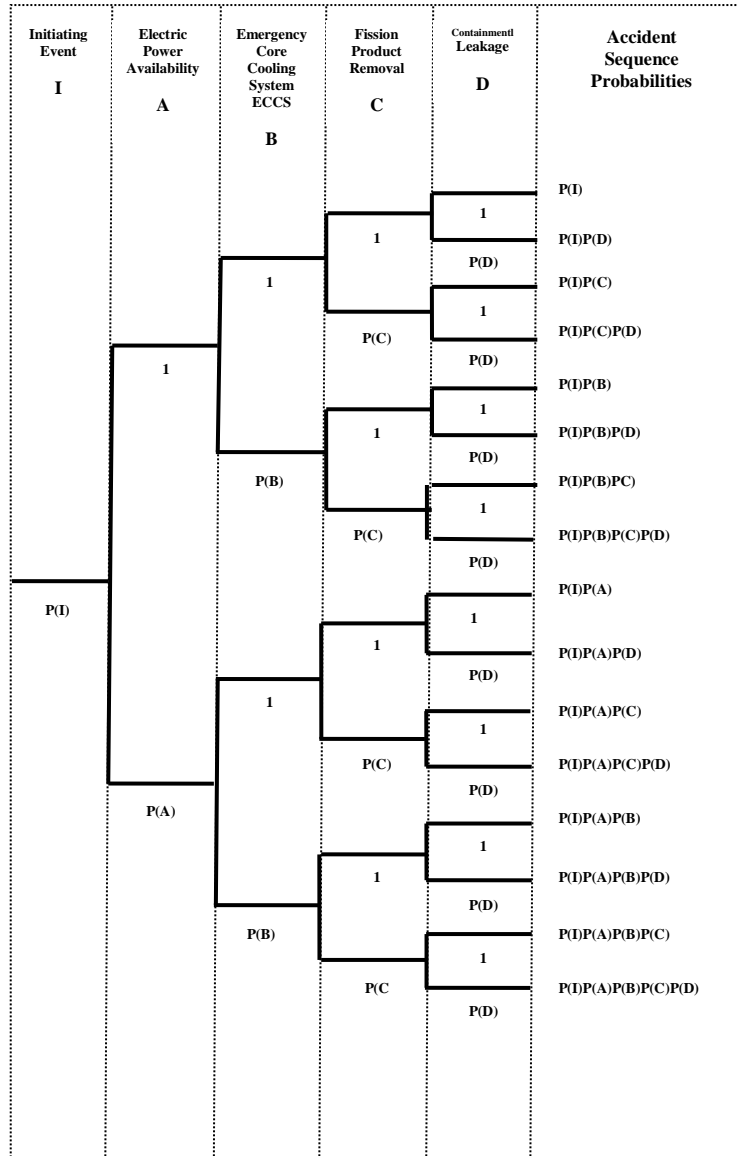


Figure 7. Basic or initial Event Tree for the Loss Of Coolant Accident, LOCA using the small probabilities approximation.

## PRUNED OR REDUCED EVENT TREES

For a binomial Event Tree with  $n$  components the number of branches in the tree  $N$  is given by:

$$N = 2^n \tag{11}$$

For  $n = 4$ , the number of branches  $N = 2^4 = 16$  branches. This number grows to be a large number for a greater number of components  $n$ , causing a combinational explosion, and making the analysis of the system at hand intractable.

Hence it becomes necessary to prune the Event Tree in to a reduced one where the illogical branches are eliminated. For instance, in the case of the “Station Blackout Accident,” both the onsite and offsite sources of power to the plant are considered as unavailable. The result is that none of the other safety systems can operate and none of the lower branches of the tree are existent. Such a pruned or reduced Event Tree is shown in Figs. 8 and 9 for a probabilistic analysis or a possibilistic analysis respectively.

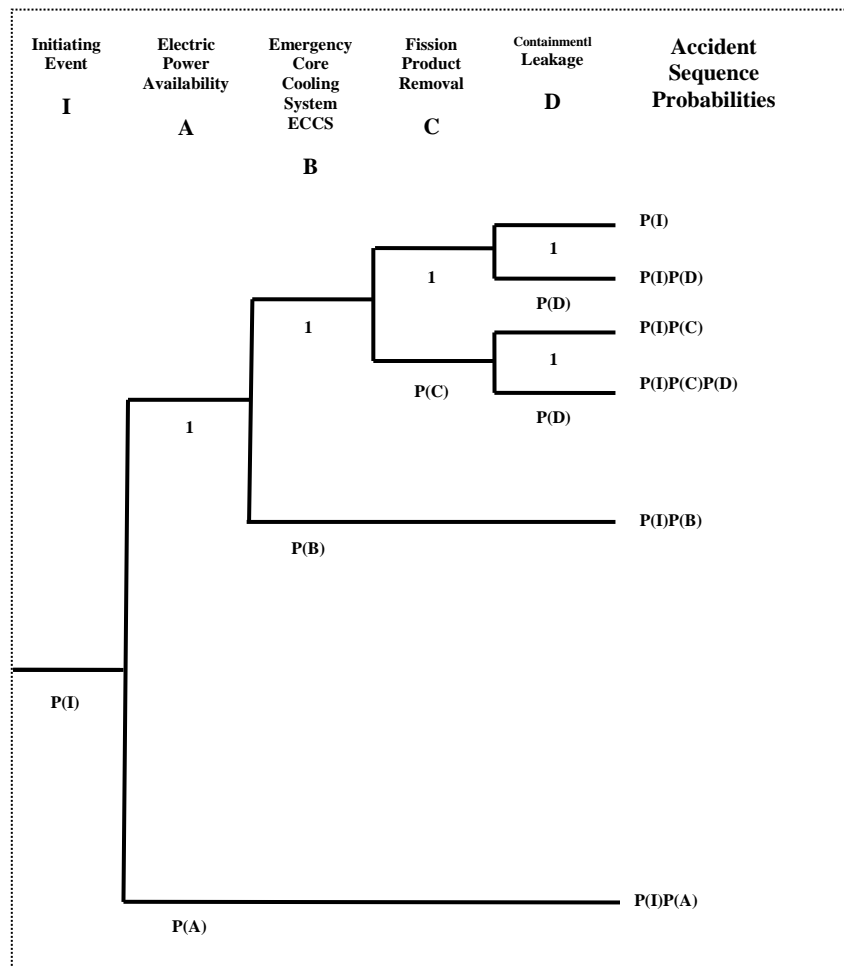


Figure 8. Reduced or pruned probabilistic Event Tree for the LOCA accident.

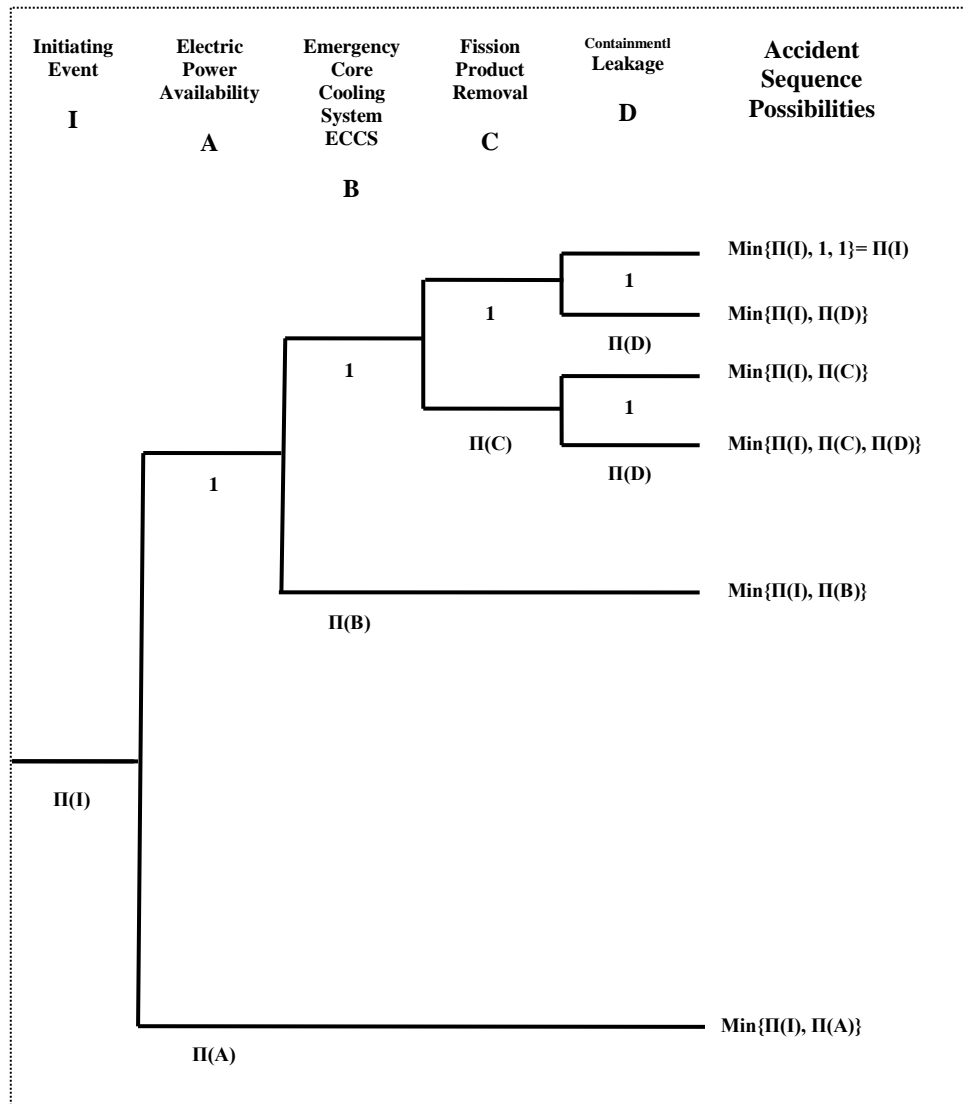


Figure 9. Reduced or pruned tree for the possibilistic analysis of a LOCA accident.

## EVENT TREE CONSTRUCTION

The steps involved in Event Tree Analysis are shown in Fig. 9. The basic or initial tree is constructed by defining the initial events, identifying the relevant components composing the system, and enumerating the success and failure states of each component in the system.

The pruned or reduced tree is then constructed by accounting for timing and the sequential and conditional dependencies.

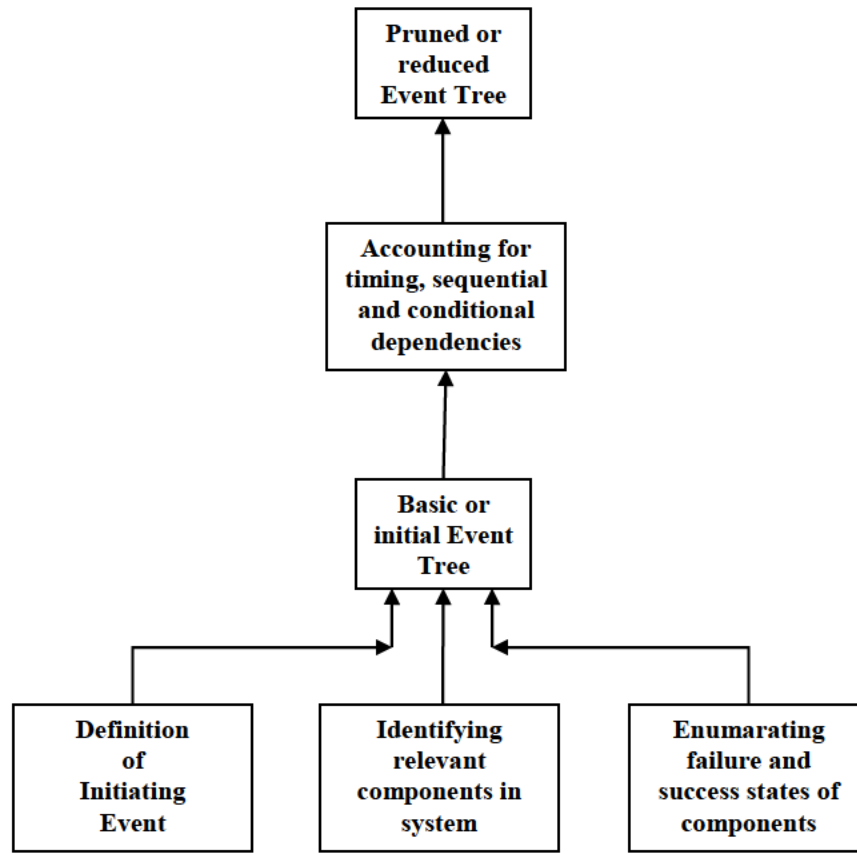


Figure 10. Tasks involved in the pruned or reduced Event Tree construction.

The failure probabilities or possibilities are generated from Fault Trees that describe the top event estimating the failure probability or possibility of each component in the system of interest.

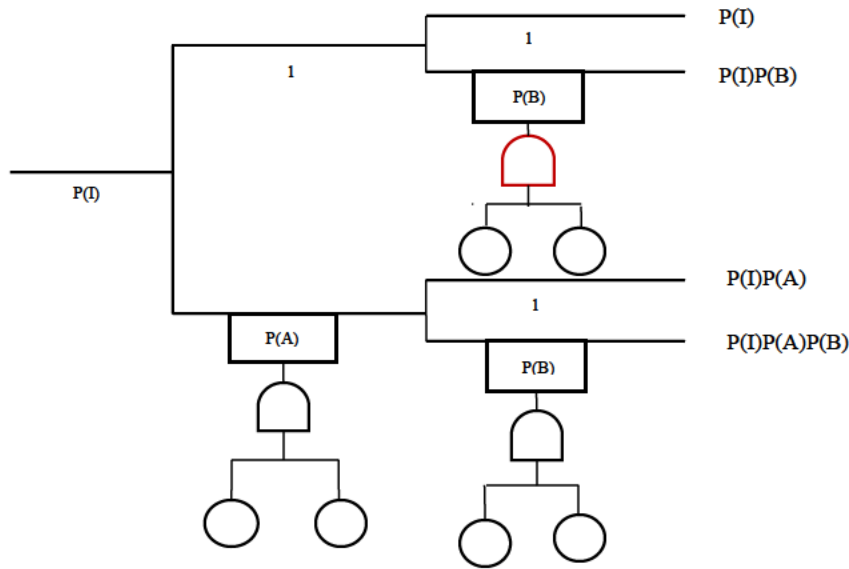


Figure 11. Coupling of Fault Tree Analysis to Event Tree Analysis.

## COMPARISON OF FAULT TREE ANALYSIS AND EVENT TREE ANALYSIS

Fault Tree Analysis, according to logic uses deductive reasoning, or backward – chaining, moving from the general to the specific. Event Tree analysis, on the other hand uses logical induction to move from the specific to the general, or forward chaining. Event Tree Analysis is also designated as a data-driven system,

The two methodologies are complementary to each other in problem-solving, even though each can be used separately under different situations.

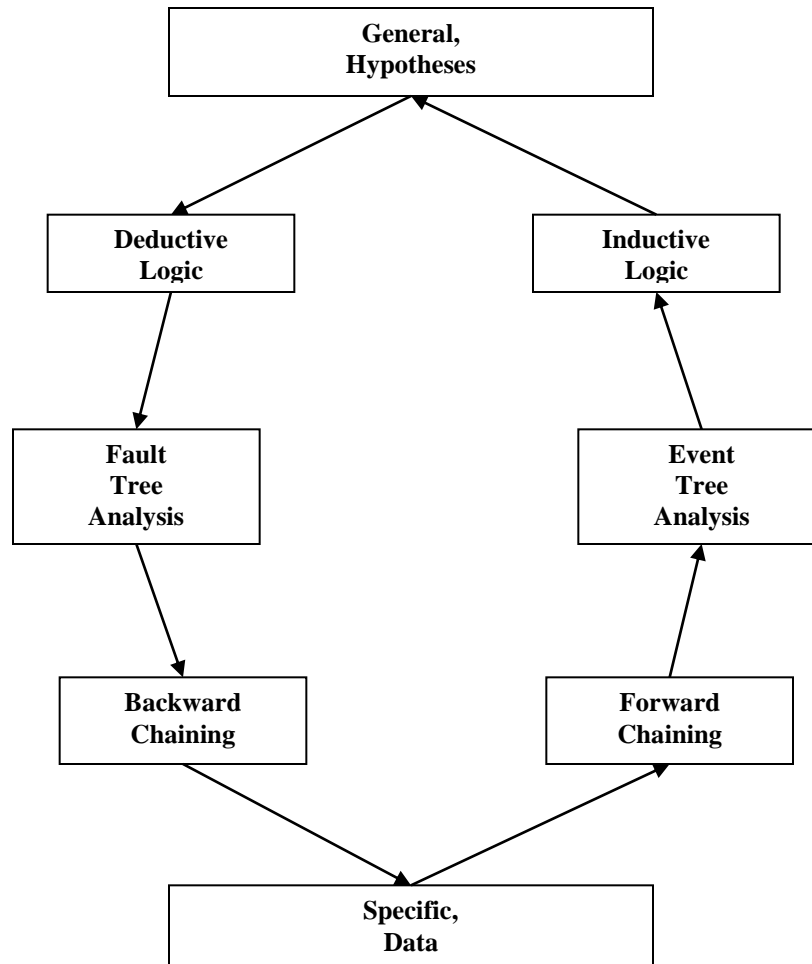


Figure 12. Fault Tree and Event Tree Analysis from the logic perspective.

## EXERCISE

1. An initiating event for an accident occurs with a probability  $P(I)=10^{-3}$ . To mitigate that type of accident the system was designed with three Engineered Safety Features (ESFs). The probabilities of failure of these ESFs are:  $P(A) = 10^{-2}$ ,  $P(B) = 10^{-3}$ , and  $P(C) = 10^{-4}$ .

a. Construct the event tree that describes this system.

b. Using the small probabilities approximation, calculate the probabilities of failure for each of the different accident sequences in the Event Tree.

c. If we consider that we have a possibilistic rather than a probabilistic Event Tree, calculate the possibilities for the different accident sequences for:

$$\pi(I) = 10^{-3}, \pi(A) = 10^{-2}, \pi(B) = 10^{-3}, \pi(C) = 10^{-4}.$$