GAMMA RAYS INTERACTION WITH MATTER

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INTRODUCTION

Gamma rays interaction with matter is important from the perspective of shielding against their effect on biological matter. They are considered as ionizing radiation whose scattering by electrons and nuclei leads to the creation of a radiation field containing electrons and ions. The main modes of interaction of gamma rays with matter are the photo effect both in its photoelectric and photonuclear forms, Compton scattering and electron positron pair production. To a minor extent Rayleigh scattering and Thomson scattering also occur.

Each of these processes occurs in different forms. Different types of scattering can occur depending on the quantum mechanical properties of the gamma photons. Electron positron pairs can be formed in the field of a nucleus and in that of an electron. The photoelectric effect can knock out atomic electrons, whereas the photonuclear reaction would knock out elementary particles from the nucleus.

Gamma rays are emitted in the decay process of radioactive isotopes. On a cosmic scale, Gamma Ray Bursts (GRBs) of magnetars generate intense gamma radiation fields that could affect space exploration. In addition, bursts of Terrestrial Gamma Ray Flashes, TGFs occur relatively high in the atmosphere as a result of thunderstorms and are not from the same sources of gamma rays seen on the ground. About 15 to 20 events are observed per month.

GAMMA PHOTONS ENERGY

A particle of zero rest mass such as a neutrino or a gamma photon will have a kinetic energy given by:

$$E_\gamma = h \nu,$$

where:

- $h$ is Planck’s constant $= 6.62 \times 10^{-27} [\text{erg.sec}]$,
- $\nu = \frac{c}{\lambda}$ is the frequency of the gamma photon,
- $c = 3 \times 10^{10} [\text{cm/sec}]$ is the speed of light,
- $\lambda$ is the wave length of the electromagnetic radiation [cm].

The momentum carried by the gamma photon is a vector quantity given by:

$$\vec{p}_\gamma = \frac{h \nu}{c} = \frac{E_\gamma}{c}$$
Gamma rays interaction with matter causes the generation of other charged particles such as positrons and electrons at relativistic speeds. If we consider the ratio of the particle speed to the speed of light as:

$$\beta = \frac{v}{c},$$

and its rest mass as \(m_0\), then the particle’s relativistic parameters become:

\[
Mass = m = \frac{m_0}{\sqrt{1 - \beta^2}}
\]

\[
Momentum = p = m\beta = \frac{m_0\beta c}{\sqrt{1 - \beta^2}}
\]

\[
Kinetic\, energy = T = m_0c^2 \left( \frac{1}{\sqrt{1 - \beta^2}} - 1 \right) = mc^2 - m_0c^2
\]

\[
Total\, Energy = E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \beta^2}}
\]

Squaring and rearranging Eqn.3, we can obtain a relationship between total energy \(E\) and momentum \(p\):

\[p^2 = m^2c^2 - m_0c^2 = m^2v^2\]

Dividing by \(m_0c^2\), we get:

\[
\left( \frac{p}{m_0c} \right)^2 = \left( \frac{mc}{m_0c} \right)^2 - 1 = \left( \frac{E}{m_0c^2} \right)^2 - 1
\]

Rearranging this equation yields:

\[E^2 = (m_0c^2)^2 + p^2c^2\]  

**PHOTOELECTRIC EFFECT**

In the photoelectric process a gamma photon interacts with an orbital electron of an atom. The electron receives kinetic energy from the gamma photon and is knocked out of its orbit. The vacancy created is promptly filled by one of the outer electrons, whose transition is accompanied by the emission of characteristic soft electromagnetic radiation in the x-rays ultraviolet, or visible regions of the electromagnetic spectrum.
The gamma photon energy is shared among the kinetic energy of the knocked out electron and the characteristic transition radiation according to the conservation of energy equation:

\[ E_\gamma = E_e + E_a + E_B, \]  

where 
- \( E_\gamma \) is the initial gamma photon kinetic energy,
- \( E_e \) is the kinetic energy acquired by the knocked out electron,
- \( E_a \) is the kinetic energy of the recoiling atom,
- \( E_B \) is the binding energy of the electron in the atom, equal to the excitation energy of the atom after electron ejection.

for \( K \) electrons \( E_B = 13.6(Z-1)^2 \text{ eV} \),

The recoil atom kinetic energy is of the order of:

\[ (m_e/M)E_e, \]

where \( M \) is the mass of the atom,
\( m_e \) is the mass of the electron.

Since:

\[ (m_e/M) = 10^{-4}, \]

the recoil energy of the atom can be neglected in Eq. 8 leading to:

\[ E_e = E_\gamma - E_B = h\nu - E_B, \]  

\( \nu \)
Conservation of momentum also applies:

\[ \overline{p}_\gamma = \overline{p}_e + \overline{p}_a \]  

For gamma rays energies above 0.5 MeV, photoelectrons are mostly ejected from the K shell of an atom.

The photoelectric interaction cross section is inversely dependent on the gamma photon energy as well on the atomic number, or the number of electrons in the element it is interacting with. An empirical relation can be written in the form:

\[ \sigma_{pe} = \frac{CZ^n}{(h\nu)^m} \approx \frac{CZ^5}{(h\nu)^{3.5}}, \]  

where m ranges from 1 to 3, and n ranges from 4 to 5. This implies that the photoelectric interaction cross section is large for elements of high atomic number, and increases with decreasing gamma ray energy as shown in Fig. 2. Gamma ray photons that have been degraded in energy by the process of Compton Scattering subsequently undergo photoelectric absorption.

Fig. 2: Gamma rays mass attenuation coefficients in Lead (Pb), showing the contributions from the photoelectric effect, Compton scattering, and pair production.

The photoelectric process is always accompanied by a secondary emission since the atom cannot remain indefinitely in an excited state, thus:
1. The atom emits X-rays and returns to the ground state.
2. Auger electrons are emitted from the outer electronic shells carrying out the excitation energy. This secondary radiation is also later absorbed and occurs in scintillators used in gamma rays detection.

**PHOTONUCLEAR EFFECT**

Nucleons are bound in most nuclei with an energy ranging from 6 to 8 MeV. Thus photons having energies less than 6 MeV cannot induce many nuclear reactions. No radioactive processes except for a few short-lived low-Z nuclides such as N\(^{16}\), as shown in Table 1 have energies that high. These energetic gammas exclude access to parts of the turbine hall in Boiling Water Reactors. Since they have a short half-life they are routed to the top of the reactor, then to the bottom of the building, before being introduced to the turbine. The transit time is sufficient to eliminate much of their radioactivity as \(\gamma\)N\(^{16}\) decays into \(\gamma\)O\(^{16}\) through negative beta decay.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Energy (MeV)</th>
<th>Half-life</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)N(^{16})</td>
<td>6.129</td>
<td>7.10 s</td>
</tr>
<tr>
<td>81TI(^{208})</td>
<td>2.6148</td>
<td>3.053 m</td>
</tr>
<tr>
<td>11Na(^{24})</td>
<td>2.754</td>
<td>15.02 h</td>
</tr>
<tr>
<td>89Ac(^{228})</td>
<td>0.9112</td>
<td>6.13 h</td>
</tr>
</tbody>
</table>

Table 1: Energetic gammas emitting isotopes

Reactions produced with such sources are therefore excitations of the nuclei to isomeric levels and the photodisintegration of the deuteron, with a threshold of 2.23 MeV, is such an example:

\[ \gamma + D^2 = H^1 + n^1, \]  \hspace{1cm} (12)

where the energetic gamma photon is capable of splitting the deuteron nucleus into its constituent proton and neutron.

Another photonuclear reaction is the photo-disintegration of the Be\(^9\) isotope with a lower threshold energy of 1.67 MeV:

\[ \gamma + Be^9 = H^4 + He^4, \]  \hspace{1cm} (13)

\[ 4Be^8 = 2He^4 + 2He^4. \]
In this reaction the Be\textsuperscript{8} product is unstable and disintegrates within 10\textsuperscript{-14} sec into two helium nuclei.

These reactions can be initiated using electrons for known energy to produce external bremsstrahlung radiation for dissociating the deuteron or beryllium.

Since the lighter elements have large nuclear level spacing, very energetic gamma rays can be emitted, and then used to induce photonuclear reactions. With accelerators operating at a moderate high voltage of 500 to 1,000 keV, high intensities gamma rays at 10\textsuperscript{6} photons/sec can be generated, as shown in Table 2.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Gamma ray energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( _1^1\text{H} \rightarrow _8^4\text{Be} + \gamma )</td>
<td>14.8, 15.0, 17.6</td>
</tr>
<tr>
<td>( _1^1\text{H} + _1^3\text{T} \rightarrow _8^2\text{He} + \gamma )</td>
<td>19.8 + 0.75E\text{p} *</td>
</tr>
</tbody>
</table>

\*E\text{p} is the proton’s energy.

Photonuclear reactions can be used to produce neutron sources which can be used in a variety of applications such as nuclear medicine and radiography. Table 3 lists such possible sources.

<table>
<thead>
<tr>
<th>Source Composition</th>
<th>Reaction</th>
<th>Q value (MeV)</th>
<th>Neutron Energy (MeV)</th>
<th>Neutron yield (per 10\textsuperscript{6} disintegrations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ra + separate Be</td>
<td>( \gamma + _8^4\text{Be} \rightarrow _8^4\text{Be} + \gamma )</td>
<td>-1.67</td>
<td>&lt;0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>Ra + separate D\textsubscript{2}O</td>
<td>( \gamma + _1^1\text{H} \rightarrow _8^4\text{Be} + \gamma )</td>
<td>-2.23</td>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>Na\textsuperscript{24} + Be</td>
<td>( _8^4\text{Be} \rightarrow _8^2\text{He} + _8^2\text{He} )</td>
<td>-1.67</td>
<td>0.8</td>
<td>3.8</td>
</tr>
<tr>
<td>Na\textsuperscript{24} + D\textsubscript{2}O</td>
<td>( _8^4\text{Be} \rightarrow _8^2\text{He} + _8^2\text{He} )</td>
<td>2.23</td>
<td>0.2</td>
<td>7.8</td>
</tr>
<tr>
<td>Y\textsuperscript{88} + Be</td>
<td>( _8^4\text{Be} \rightarrow _8^2\text{He} + _8^2\text{He} )</td>
<td>-1.67</td>
<td>0.16</td>
<td>2.7</td>
</tr>
<tr>
<td>Y\textsuperscript{88} + D\textsubscript{2}O</td>
<td>( _8^4\text{Be} \rightarrow _8^2\text{He} + _8^2\text{He} )</td>
<td>-2.23</td>
<td>0.3</td>
<td>0.08</td>
</tr>
<tr>
<td>Sb\textsuperscript{124} + Be</td>
<td>( _8^4\text{Be} \rightarrow _8^2\text{He} + _8^2\text{He} )</td>
<td>-1.67</td>
<td>0.02</td>
<td>5.1</td>
</tr>
<tr>
<td>La\textsuperscript{140} + Be</td>
<td>( _8^4\text{Be} \rightarrow _8^2\text{He} + _8^2\text{He} )</td>
<td>-1.67</td>
<td>0.6</td>
<td>0.06</td>
</tr>
<tr>
<td>La\textsuperscript{140} + D\textsubscript{2}O</td>
<td>( _8^4\text{Be} \rightarrow _8^2\text{He} + _8^2\text{He} )</td>
<td>-2.23</td>
<td>0.15</td>
<td>0.2</td>
</tr>
<tr>
<td>Ac\textsuperscript{228} + Be</td>
<td>( _8^4\text{Be} \rightarrow _8^2\text{He} + _8^2\text{He} )</td>
<td>-1.67</td>
<td>0.8</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Energetic gamma photons are emitted from daughters nuclides in the thorium decay chain, such as the 2.6146 MeV of energy gamma ray photon emitted by $^{81}$Thallium$^{208}$, whose half-life is 3.053 minutes. This energy exceeds the binding energy of the deuteron at 2.23 MeV, and can lead to its disintegration. The presence of thorium and its daughters with deuterium in ordinary or heavy water, would lead to a source of energy from the photonuclear reaction in Eqn. 6. Elemental transmutations can also be expected from the presence of neutrons and protons. This suggests that the process of nucleo synthesis is occurring here on earth, and not just in the stars. This topic has not been investigated, and could be the source of transmutations in experiments thought to be cold fusion experiments.

The excitation functions for some simple processes such as $(\gamma, n)$ and $(\gamma, p)$ reactions and some $(\gamma, 2n)$ and photo fission $(\gamma, \text{fission})$ reactions rise with increasing photon energy, then drop again without an increase in the cross section for competing reactions. The total cross section displays a “giant resonance” behavior. It can be ascribed to the excitation of dipole vibrations of all the neutrons in the nucleus moving collectively against all the protons. The energy of the resonance peak decreases with increasing mass number $A$. It is 24 MeV for $^{16}$O$^{16}$, and 14 MeV for $^{181}$Ta$^{181}$. With gamma rays energy exceeding 150 MeV, such as those generated by cosmic rays, meson production occurs and leads to intra nuclear cascades, spallation and high energy fission.

**PHOTOFISSION OF NUCLEI**

If high-energy protons bombard fluorite or CaF$_2$, gamma photons of 6.3 MeV energy can be produced. These can make the nuclei of uranium and thorium so unstable, that they can fission. High energy X rays of 8-16 MeV energy produced by particle accelerators such as the betatron can also cause uranium fission. The threshold energy as shown in Table 4 does not vary much from one nuclide to the other in the thorium and uranium area of mass numbers. However even 16 MeV photon cannot induce fission of lead.

<table>
<thead>
<tr>
<th>Photofission Threshold (MeV)</th>
<th>Nuclide</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.40</td>
<td>$^{90}$Th$^{230}$</td>
</tr>
<tr>
<td>5.18</td>
<td>$^{92}$U$^{233}$</td>
</tr>
<tr>
<td>5.31</td>
<td>$^{92}$U$^{235}$</td>
</tr>
<tr>
<td>5.08</td>
<td>$^{92}$U$^{238}$</td>
</tr>
<tr>
<td>5.31</td>
<td>$^{94}$Pu$^{239}$</td>
</tr>
</tbody>
</table>

**COMPTON SCATTERING**

This is the most dominant process of gamma rays interaction with matter. A gamma ray photon collides with a free electron and elastically scatters from it as shown in Fig. 3. Energy and momentum cannot be conserved if a photon is completely absorbed by a free electron at rest.
Moreover electrons in matter are neither free nor at rest. However, if the incident photon energy is much larger than the binding energy of the electron, which is its ionization potential in gases or work function in a solid, and if the incident photon momentum is much larger than the momentum of the interacting electron, then we can approximate the state of the electron in a simple model as free and at rest. In this case a gamma ray can interact with a loosely bound electron by being scattered with an appropriate loss in energy.\(^2\)

The total energy of a relativistic particle related to its momentum is from Eqn. 7:

\[
E = \left[ (m_0c^2)^2 + p^2c^2 \right]^{1/2}
\]  

(14)

Denoting the energy of the initial gamma photon as \(E_\gamma\), and after collision as \(E'_\gamma\) and scattering through an angle \(\theta\) as shown in Fig. 3, and applying the relativistic conservation of energy and of momentum for such an elastic collision yields:

Conservation of energy:

\[
E_\gamma + E_0 = E'_\gamma + (E_0^2 + c^2p^2)^{1/2}
\]

(15)

Conservation of momentum:

\[
\frac{E_\gamma}{c} = \frac{E'_\gamma}{c} + \vec{p}
\]

(16)

where \(E_0 = m_0c^2\) is the total energy of the electron when it is at rest = 0.511 MeV. \(m_0\) is the mass of the electron.

The vector equation describing conservation of momentum can be expanded along the incident photon path and perpendicular to it as:

\[
\frac{E_\gamma}{c} = \frac{E'_\gamma}{c} \cos \theta + p \cos \varphi
\]

(17)

\[
0 = -\frac{E'_\gamma}{c} \sin \theta + p \sin \varphi
\]

(18)

Eliminating the angle \(\varphi\) using the relationship:

\[
\cos^2 \phi + \sin^2 \varphi = 1,
\]

yields:

\[
p^2c^2 = E_\gamma^2 - 2E_\gamma E'_\gamma \cos \theta + E'_\gamma^2
\]

(19)

Substituting the value of \(p^2c^2\) into Eqn. 15, squaring both sides, and canceling the equal terms yields an expression for the outgoing photon energy as:
The last equation can be expressed as the following wave shift relationship:

\[ \Delta \lambda = \lambda' - \lambda = \lambda_0 (1 - \cos \theta) \]  

(21)

where:

\[ \lambda_0 = \frac{\hbar}{m_0 c} = 2.42621 \times 10^{-10} \text{ [cm]}, \]

is the Compton wave length of the electron,

\( m_0 \) is the electron mass,

\( \lambda \) and \( \lambda' \) is the wave length of the gamma photons before and after scattering,

\( \theta \) is the scattering angle of the gamma photon.

It is interesting to notice that the wavelength shift is independent of the incident gamma ray energy.

For a given incident photon energy, there exists a minimum energy, corresponding to a maximum wavelength for the scattered gamma photon when it is scattered in the backward direction at \( \theta = 180^\circ \). In this case, \( \cos \theta = -1 \), and:

\[ (E'_\gamma)_{\text{min}} = \frac{E_0/2}{1 + E_0/2E_\gamma} \]  

(22)

For large gamma photons energies the minimum energy of the gamma photon approaches \( E_0/2 = 0.25 \text{ MeV} \).

Also, for high energy gamma rays, from Eqn. 20, we get:
\[ E' \gamma \approx \frac{m_0c^2}{1 - \cos \theta} \]  

for all scattering angles \( \theta \) except near \( 0^0 \).

The probability of the Compton effect is proportional to the number of electrons in the atom, therefore:

\[ \sigma_c \approx const.Z \]  

**POSITRON ELECTRON PAIR PRODUCTION**

A photon of at least 1.02 MeV or the equivalent of two electrons masses \((2m_0c^2)\) can create an electron positron pair. In empty space, momentum and energy cannot be conserved. In the vicinity of a nucleus, the process is possible since the nucleus can carry some momentum and energy. Figure 4 shows the formation of an electron positron pair from an energetic gamma photon in a cloud chamber. A magnetic field curves the particles paths in different directions because of their opposite charges. Some Compton and photoelectric electrons are released when the incoming gamma photon penetrates the chamber wall.

Taking the square root of Eqn. 7 for a relativistic particle yields:

\[ E = \pm \left[ (m_0c^2)^2 + p^2c^2 \right]^{\frac{1}{2}} \]

**Fig. 4: Electron positron pair produced in a cloud chamber by high energy gamma rays.**
It was shown by Dirac that the ambiguity of the sign in Eqn. 25 is not a mathematical accident. The positive energies $E$ represent a particle of rest mass $m_0$ and momentum $p$, and the negative energy states represent a particle of rest mass $-m_0$ and momentum $-p$ as shown in Fig. 5.

No particles can occupy the energy interval:

$$+m_0c^2 \geq E \geq -m_0c^2.$$

Nature is such that all negative energy states are filled with electrons in the absence of any field or matter, and no effect of these electrons is noticeable in the absence of any field or matter. If an electron is ejected from a negative energy state by action of a gamma photon, a hole is formed in the negative energy states like a bubble is formed in a liquid as it is being heated.

The hole in the negative energy states means that the system acquires a mass:

$$-(-m_0) = +m_0,$$
a momentum:

$$-(-p) = +p,$$
and a charge:

$$-(-e) = +e.$$

This bubble or hole corresponds then to a positron of mass $+m_0$, momentum $p$, and charge $+e$.

When the bubble is created an electron also appears in a positive energy state with kinetic energy $E_e$. Conservation of energy requires that:

$$E_\gamma = h\nu = E_e + E_p + 2m_0c^2. \quad (25)$$

This equation can be satisfied in the vicinity of a third particle or a nucleus which can take the excess momentum. If the nucleus does not take much momentum, then the minimum energy for pair production occurs when:

$$E_e + E_p = 0,$$

And consequently, the minimum energy for pair production becomes:

$$(E_\gamma)_{\text{min}} = (h\nu)_{\text{min}} = 2m_0c^2 = 1.02 \text{ MeV}. \quad (26)$$

The probability of the process or its cross section increases with increasing photon energy and atomic number $Z$, as shown in Fig. 2. In particular, it is proportional to the square of the atomic number as:
Pair production is almost always followed by the annihilation of the positron, usually leading to the emission of two 0.51 MeV gamma photons. A single photon is emitted in rare instances where the positron energy is very small, so that a neighboring atom can take the available momentum.

Fig 5: Formation of an electron-positron pair.

A positron and an electron can also form a positronium, an atom like structure in which each one of the particles moves about their common center of mass. It is short-lived depending on the spin orientation of the particles with $10^{-10}$ or $10^{-7}$ sec lifetime, after which they annihilate each other.

RAYLEIGH SCATTERING
Fig. 6: Interaction of a gamma photon with a bound electron: Rayleigh scattering.

If the gamma photon is scattered by a bound electron that is not removed from its atom then Eqns. 8 and 10 still hold. This occurs with the momentum and kinetic energy of the entire recoiling atom replacing that of the electron. Thus in the wave shift Eqn. 21, the mass of the electron must be replaced by the mass of the entire atom. This process shown in Fig. 6 is called Rayleigh scattering, and its wavelength shift is practically negligible.

Rayleigh scattering increases with the atomic number $Z$ of the scatterer material, since the binding energy of the inner electrons is proportional to $Z^2$ implying that an increasing fraction of the atomic electrons is considered as bound. The radiation scattered from all bound electrons in one atom interferes coherently and Rayleigh scattering is peaked around $\theta = 0$.

THOMSON SCATTERING

Gamma radiation can scatter on a nucleus with or without excitation of the nucleus. In Thomson scattering, gamma radiation can scatter on the nucleus without excitation. This process interferes coherently with Rayleigh scattering but occurs with a much lower probability.

ABSORPTION OF GAMMA RAYS IN MATTER

The atomic cross section for the three main processes: the photoelectric process, Compton scattering, and pair production increase with increasing $Z$. For this reason, heavy elements are much more effective for gamma radiation than light elements. Lead and Uranium can be used to shield against gamma rays. Because the photoelectric effect and Compton scattering decrease, and pair production increase with increasing energy, the total absorption in a given element has a minimum, or maximum transparency at some energy. This is also a window, through which gamma radiation would leak from a given shield as shown in Fig. 2 and Table 5.

The total gamma ray interaction cross section of a substance can be represented as:

$$\sigma_t = \sigma_C + \sigma_{pe} + \sigma_{pp}$$  \hspace{1cm} (28)

Table 5: Transparency window for different gamma ray shielding materials

<table>
<thead>
<tr>
<th>Shielding material</th>
<th>Transparency window (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead</td>
<td>3</td>
</tr>
<tr>
<td>Copper</td>
<td>10</td>
</tr>
<tr>
<td>Aluminum</td>
<td>22</td>
</tr>
</tbody>
</table>

If we assume that each interaction event leads to the removal of a gamma ray photon from a parallel gamma ray beam, we can represent the attenuation of the beam by a layer of material of thickness $x$ [cm] as follows:

$$I(x) = I_0 e^{-N\sigma x}$$  \hspace{1cm} (29)
where the number density \( N' \), or number of atoms or nuclei in 1 cm\(^3\) of material of the material is given by the modified form of Avogadro’s law as:

\[
N' = \frac{\rho A_v}{M}
\]

(30)

where \( \rho \) is the density of the material in [gm/cm\(^3\)], \( M \) is the molecular or atomic weight of the material in atomic mass units [amu].

The attenuation coefficient for gamma rays is defined as:

\[
\mu = N' \sigma_i,
\]

(31)

Consequently Eqn. 29 can be written as:

\[
I(x) = I_0 e^{-\mu x}
\]

(29)'

The physical significance of the attenuation coefficient \( \mu \) is that it is a summation of the microscopic cross section areas in cm\(^2\) per unit volume (cm\(^3\)) of the material. It has units of \((\text{cm}^2/\text{cm}^3)\) or cm\(^{-1}\).

If we define the relaxation length giving now units of [cm] as:

\[
\lambda = 1 / \mu
\]

(32)

then a third form of Eqn. 29 can be written as:

\[
I(x) = I_0 e^{-x / \lambda}
\]

(29)''

If we further define the mass attenuation coefficient shown in Fig. 2 for lead as:

\[
\mu_m = \frac{\mu}{\rho}
\]

(33)

which is a measure of the probability of interaction of a gamma photon in a unit mass of a substance, usually taken as 1 gm. Its units are: [cm\(^2\)/gm]

In this case there is still another form of Eqn. 29 that can be written:

\[
I(x) = I_0 e^{-\mu_m x}
\]

(29)'''

Equation 29 in its different forms can be used for the calculation of gamma rays attenuation in matter if the geometry is such that any gamma photon that is scattered at even a
small angle leaves the beam, and does enter the detector. This is designated as the good geometry or narrow beam condition.

**BUILDUP FACTOR**

In practical cases, a gamma photon undergoing Compton scattering can reenter the detector in broad beam conditions. A purely exponential function cannot describe a broad beam condition. The deviation is referred to as the *buildup* of scattered gamma rays that have undergone Compton scattering and are reentering the detector.

Account is practically taken of this effect by the introduction of a *buildup factor* $B$. The value of $B$ depends on the nature and thickness of the attenuating medium and on the gamma ray energy. The build up factor is thus defined as:

$$B = \frac{\text{Actual gamma ray flux}}{\text{Flux obtained using exponential attenuation law}}$$

The calculation and choice of build up factor is a part of the field of gamma ray shielding analysis. In general the practical attenuation law for gamma rays allowing for fluxes takes the form:

$$I(x) = I_0 B(\mu x, E_\gamma) e^{-\mu(E_\gamma)x}$$  \hspace{1cm} (29)***

where $B$ is the build up factor.

**GAMMA RAY BURSTS, GRBs**

A massive blast that lasted 200 seconds was detected September 4, 2005 by the Swift satellite. Swift has detected 71 bursts since its November 2004 launch. The event occurred about 1.1 billion years after the big bang, the explosion that created the universe an estimated 13.7 billion years ago, the scientists calculated. The only more distant objects ever detected are a quasar and a single galaxy, both about 12.7 billion light-years away. Gamma ray bursts are brighter than galaxies or even quasars, distant, bright objects that scientists theorize are massive black holes that project energy by devouring stars.

An earlier powerful gamma ray burst occurred on Dec. 27, 2004. The eruption was recorded by NASA’s gamma rays Swift observatory and by the National Science Foundation's Very Large Array of radio telescopes, along with other European satellites and telescopes in Australia. The Swift satellite observatory, named Swift for its speedy pivoting and pointing was among the instruments that detected the flare. It was launched to probe the workings of black holes. The satellite, operated by the Goddard Space Flight Center in Greenbelt, is designed to detect gamma ray outbursts and quickly pivot to record them. It also recorded the afterglow of the blast.

The gamma rays hit the earth’s ionosphere and created ionization, briefly expanding it. The flash of gamma rays was so powerful that it bounced off the moon and lit up the earth's upper atmosphere. Had this happened within 10 light-years away from the earth, it would have severely damaged its atmosphere and possibly triggered a mass extinction. It would have
destroyed the ozone layer causing abrupt climate change and mass extinctions due to increased space radiation reaching the earth’s surface. Scientists wonder whether major species die offs in the past might have been triggered by closer such stellar explosions.

The gamma ray burst occurred at a neutron star called SGR 1806-20 about 50,000 light years away from the solar system. A light-year is the distance light travels in a year, about 6 trillion miles or 10 trillion kilometers. The blast was 100 times more powerful than any other similar witnessed eruption.

Gamma ray bursts are thought to occur when a star runs out of hydrogen fuel and starts to burn heavier elements produced by nuclear fusion. Eventually the star is left with only iron, which will not burn. The star collapses and, if it is big enough, creates a black hole with gravity so intense that nothing can escape from it. The event is accompanied by a spectacular gamma ray explosion.

A neutron star is the remnant of a star that was once several times more massive than the sun. When their nuclear fuel is depleted, they gravitationally collapse as a supernova. The remaining dense core is slightly more massive than the sun but has a diameter typically no more than 12 miles or 20 kilometers. Millions of neutron stars fill the Milky Way galaxy. A dozen or so are ultra-magnetic neutron stars or magnetars. The magnetic field around one is about 1,000 trillion gauss, strong enough to strip information from a credit card at a distance halfway to the Moon.

Of the known magnetars, four are called Soft Gamma Repeaters, or SGRs, because they flare up randomly and release gamma rays. The flare on SGR 1806-20 unleashed about 10,000 trillion trillion trillion watts of energy.

Fig. 7: Gamma ray burst. The aftermath of the blast is a smoldering oblong ring that glows for several days after the flare, or afterglow, caused by debris launched into the gas surrounding the star.

The flare was observed in the constellation Sagittarius or the Archer. The explosion, which lasted over a one-tenth of a second, released energy more than the Sun emits in 150,000 years.
This might be an once-in-a-lifetime event for astronomers, as well as for the neutron star. Only two other giant flares were observed in the previous 35 years, and this event was one hundred times more powerful than any of them.

SGR 1806-20 was one of only about a dozen known magnetars. These fast-spinning, compact stellar corpses, no larger than a big city, create intense magnetic fields that trigger explosions.

The naked eye and optical telescopes could not spot the explosion because it was brightest in the gamma-ray energy range. No known eruption beyond our solar system has ever appeared as bright upon arrival. The event equaled the brightness of the full Moon's reflected visible light.

The SGR 1806-20 star spins once on its axis every 7.5 seconds, and it is surrounded by a magnetic field more powerful than any other object in the universe which may have snapped in a process called magnetic reconnection. Other scientists believe the magnetic field of the magnetars can shift like an earthquake, causing it to eject a huge burst of energy.

**OCCURRENCE OF GAMMA RAY BURSTS**

Every day gamma ray bursts illuminate the sky. They come from random directions from the universe and have become the target of intense research and study by astronomers.

In 1991 NASA launched the Compton Gamma Ray Observatory carrying an instrument called the Burst and Transient Source Experiment, or "BATSE" designed specifically for the study of the enigmatic GRBs and has led to a new understanding of their origin and distribution in the universe.

Lasting anywhere from a few milliseconds to several minutes, GRBs shine hundreds of times brighter than a typical supernova and about a million trillion times as bright as the sun, making them briefly the brightest source of cosmic gamma-ray photons in the observable universe. GRBs are detected about once per day from random directions of the sky.

GRBs were for a while the biggest mystery in high-energy astronomy. They were discovered serendipitously in the late 1960s by USA military satellites which were on the look out for potential Russian nuclear testing in violation of the atmospheric nuclear test ban treaty. These satellites carried gamma ray detectors since a nuclear explosion produces gamma rays. Prompt gamma rays are emitted from the fission process, followed by delayed gamma rays from the resulting fission products.

As recently as the early 1990s, astronomers didn't even know if GRBs originated at the edge of our solar system, in our Milky Way Galaxy or far away near the edge of the observable universe. A slew of satellite observations, follow-up ground-based observations, and theoretical work have allowed astronomers to link GRBs to supernovae in distant galaxies.

From the large energy, the rapid variability and the energy spectrum of the radiation it is expected that one or more compact objects such as black holes, or neutron stars must be involved. The gamma radiation itself originates from outflowing material which expands at more than 99.995 percent of the speed of light.

**CUSPED FIELDS MODEL**

A model for GRBs must answer address two critical questions:
1. The process by which matter is extremely accelerated. Substantial energy must be distributed into a small mass to make the large velocities possible.

2. The process of radiation generation. Accelerated matter does not radiate by itself. An additional process must generate the high energy emissions.

   It is proposed that gamma ray bursts could evolve as a result of the occurrence within the magnetars of a cusped magnetic field configuration.

In most systems in which a plasma is confined by a magnetic field that surrounds it smoothly without a discontinuity, there is a tendency toward the creation of instabilities. This is so because the magnetic lines of force which are stretched around the plasma can shorten themselves by burrowing into the gas and thus force it outward.

![Fig. 8: Cusped magnetic field configuration produced by an array of four line currents alternating in direction.](image)

A confinement system which is absolutely stable against arbitrary deformations, even of finite amplitude, of the plasma can be obtained if the magnetic field lines curve away everywhere from a diamagnetic plasma. This means that the magnetic field and plasma interface is everywhere convex on the side toward the plasma. To satisfy this curvature requirement, the magnetic field must possess cusps, which are points or lines, or both, through which the magnetic field lines pass radially outward from the center of the confinement region (Fig. 8).

A laboratory geometry of this kind is called a picket fence, and consists of two layers of parallel wires carrying currents in alternating directions so that the magnetic field has a series of cusps. The magnetic fields are generated in the vicinity of the walls decreasing the power needed in maintaining them. This advantage is superseded by the more important advantage of stable confinement. Another type of cusped geometry is the biconal cusp produced by a pair of
magnetic field circular coils with the currents in them flowing in opposite directions. A succession of cusped configurations can generate a toroidal cusped system.

If there is a loss of plasma from the central volume, the lines of force would have to stretch to fill the volume previously occupied by the plasma. Since this requires an extra energy expenditure, this type of instability would not probably occur.

In the cusped configuration a singular condition exists at the center where the magnetic field goes exactly to zero. A particle passing through this point will experience a large field change occurring in a time interval less than the gyromagnetic period, particularly as this period is lengthened by the decrease in the magnetic field strength. The magnetic moment becomes no longer an adiabatic invariant of the system.

In the special case of a particle actually passing through the point where the magnetic field is zero, the particle will momentarily travel in a straight line and its motion will bear no relationship to that before its page through this point. The magnetic energy of the particle can theoretically reach infinite value along the zero magnetic field line.

If one considers a confined plasma rather than a particle, one must consider the problem of Magneto Hydro Dynamic (MHD) cumulation of energy near a zero field line. Let us consider a perfectly conducting and incompressible cylindrical plasma to be immersed in a quadrupolar steady external magnetic field:

\[ \vec{B}_e = B_0 \nabla \vec{x} \nabla \vec{y}. \]  \hspace{1cm} (34)

In the absence of currents and velocities the plasma is under an unstable equilibrium to a linear velocity perturbation:

\[ v_x = U \vec{x}, v_y = U \vec{y}. \]  \hspace{1cm} (35)

The MHD equations of motion which do not depend on the \( z \) axis variable can be exactly solved with the velocity perturbation as an initial condition.

The motion generates an axial current \( j_z \) in the \( z \) direction. Due to the pinch effect or the Lorentz force:

\[ \vec{F} = j \times \vec{B} \]  \hspace{1cm} (36)

the circular section of the cylinder is deformed into ellipses of axes \((0x, 0y)\). After a time comparable to the ratio of the initial cylinder ratio to the Alfvén velocity:

\[ V_A = \frac{B_0}{(4\pi \rho)^{1/2}} \]  \hspace{1cm} (37)

a cumulation process occurs and the elliptical cross section is stretched along the \( x \) axis or the \( y \) axis depending on the relative values of the initial perturbation velocities \( U \) and \( V \). The important result is that the plasma flattens and expands radially perpendicular to the \( z \) axis while the plasma kinetic energy and the current density increase without limit.

The MHD equations can also be solved if the plasma is considered to be a perfect gas with a finite electrical conductivity. At the finite cumulation time, the plasma initially contained
within a circular cross section plasma would be squeezed within an ellipse of zero volume. The density, velocity $V_y$, current density $j_z$, as well as the internal kinetic or magnetic energies, all become infinite inside the limiting segments.

The cumulation process requires an energy source supplied by the energy stored in the magnetic field. An interesting consequence of the cumulation process is that the electron velocity $J/n$ tends to infinity which explains the process of acceleration of particles to extremely high velocities as is observed in cosmic ray particles.

The hot plasma in the magnetic field generates high energy radiation in the form of synchrotron radiation.

**MAGNETIC FIELD ANNIHILATION MODEL**

The large acceleration and radiation and radiation can be explained by the help of a model of magnetic fields annihilation.

The annihilation process can result from a fast rotating neutron star or be generated in a fusion of a neutron star with a black hole. A compact rotating object with a magnetic field produces an electromagnetic wave comparable to an antenna.

If the plasma is expelled by the central object, in the outward traveling wave regions could contain oppositely aligned magnetic fields. Through the generation of instabilities, plasma components in which the magnetic field orientations are in opposite directions can come close to each other. As a result the magnetic field annihilates and the magnetic energy is released and transferred to the plasma. Figure 9 illustrates the principle of the field annihilation.

![Figure 9: Two dimensional sketch of the magnetic field annihilation.](image)

At the intersecting point, the magnetized plasma outflow is not only strongly accelerated but also heated up.

**ENERGETIC RADIATION IN THUNDERSTORMS**

In the physics of thunderstorms and lightning large electric fields initiate lightning and affect its propagation. Enormous bursts of energetic radiation occur in thunderstorms in the form
of electrons, x and gamma rays caused by strong electric fields in the air. These bursts generate runaway electrons that rapidly discharge the electric fields.

Runaway electrons are produced in air when the energy gained from the electric field exceeds the loss from collisions. This allows the electrons to accelerate to relativistic energies. An avalanche of such runaway electrons develops when energetic knock off electrons are produced by hard elastic or Moeller scattering with electrons in the air molecules. The knock off electrons subsequently run away producing more energetic knock off electrons and so on providing positive feedback to the avalanche.

The runaway electrons in the avalanche produce large quantities of ionization and x and gamma rays through bremsstrahlung interactions with air. Gamma rays here are of atomic not nuclear origin and are just highly energetic x rays.

The high energy gamma rays generate a small number of positrons through the process of pair production, some of which also run away, but in the opposite direction of the electrons in the Earth’s magnetic field. These then interact with antimatter ordinary electrons producing annihilation radiation gamma rays.

Bremsstrahlung gamma rays are produced when the runaway electrons collide with air. Additional seed electrons are generated by two feedback mechanisms producing more runaway breakdown. These feedback mechanisms include positrons feedback and gamma ray feedback through Compton scattering or the photoelectric effect.

The existence of runaway breakdown correlates with recent observations of energetic radiation associated with natural and triggered man made lightning experiments, thunderstorms, and red sprites.

TERRESTRIAL GAMMA RAYS FLASHES, TGFs

Using the Compton Gamma Ray Observatory, CGRO, bursts of Terrestrial Gamma Ray flashes, TGFs were first observed in 1994. The TGFs occur relatively high in the atmosphere and are not from the same sources of gamma rays seen on the ground. About 15 to 20 events are observed per month.

Prior to this time, it was thought that bursts of gamma rays had an astrophysical origin only. The photon energies exceeding 1 MeV suggest that the gamma rays are produced by the bremsstrahlung radiation from accelerated high energy electrons, since this type of radiation is emitted when electrons are scattered by nuclei. The upward beams of runaway electrons follow the earth’s magnetic field, and are thought to be accelerated by thundercloud fields. These flashes are considered to be the most energetic natural phenomenon on Earth.
Further observations by the Reuven Ramaty High Energy Solar Spectroscopic Imager, RHESSI satellite in 2005 showed that these flashes are common and that the photon energies can reach 20 MeV. Seed electrons at relativistic energies above 1 MeV are accelerated in an electric field, followed by electrical breakdown as a result of their collisions with the air molecules. These relativistic runaway breakdowns can proceed at much lower electric fields than the conventional air breakdowns in which the ambient thermal electrons are accelerated to energies sufficient to ionize nitrogen in the air. A runaways beam of 100 keV to 10 MeV would radiate gamma rays at altitudes of 30 to 70 kms. For the electrons with an energy exceeding 500 keV, the loss in energy due to scattering is insignificant above 70 kms because of the low air density. Thus most of the particles must escape into the radiation belts resulting in an injected beam with a fluence of $10^6$ to $10^7$ [electrons/cm$^2$]. In cloud to ground discharges the transverse scale of the beam is from 10 to 20 kms. For cloud to cloud discharges, it may be as large as 100 kms. The beam duration is just about 1 millisecond.

Transient intense electric fields associated with thunderclouds create a total potential drop at an altitude between 20 to 80 kilometers of more than 30 mega volts, MV for large positive cloud to ground discharges. These strong electric fields produce nonlinear runaway avalanches of accelerated electrons which collide with the air molecules stripping in the process an even larger number of relativistic electrons. A large number of relativistic electrons spreads over a large region generating the TGFs at an altitude of 30 to 70 kms.

The intense upward moving electrons enter the Earth’s Van Allen radiation belts of charged particles trapped in the Earth’s magnetic field. Some of them may become trapped there and then discharged in conjugate regions of the Earth where intense lightning discharges occur such as in the Southern Hemisphere from a Northern Hemisphere thunderstorm. There, they interact with the air molecules in the denser atmosphere creating optical emissions and x rays.
The electrons could drift around the planet and could precipitate into the atmosphere near the South Atlantic anomaly where the Earth’s magnetic field exhibits a minimum.

It is not clear what the dose to persons on the ground from these TGFs is. Experimental and theoretical investigations of the gamma ray dose effects of the TGFs during thunderstorms needs to be undertaken. If water in the clouds is dissociated into oxygen and hydrogen, the accelerated protons could be emitted and they could interact through nuclear reaction with nitrogen in the atmosphere, for instance through the \((p, n)\) reaction:

\[
\begin{align*}
^1H^1 + ^7N^{14} &\rightarrow ^6H^1 + ^8O^{14} \\
^8O^{14} &\rightarrow +_1e^0 + ^1N^{14} + \gamma (2.313\text{MeV})
\end{align*}
\]

which can also lead to the generation of gamma photons, albeit from a nuclear rather than an electronic process:

\[
+_1e^0 + \gamma &\rightarrow \gamma (0.51\text{MeV}) + \gamma (0.51\text{MeV})
\]

The neutrons can also interact with other isotopes in the atmosphere, such as through the \((n, p)\) reaction:

\[
^0n^1 + ^7N^{14} \rightarrow ^1H^1 + ^6C^{14}
\]

which would contribute to the creation of carbon\(^{14}\) in the Earth’s atmosphere in addition to the production from cosmic ray neutrons. If taken into account it would affect the results of carbon dating by suggesting a higher level of equilibrium C\(^{14}\) concentration in the Earth’s atmosphere.

REFERENCES