TWO GROUP DIFFUSION THEORY FOR BARE AND REFLECTED REACTORS

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4/27/2006

INTRODUCTION

In the two group theory treatment we consider a thermal energy group, and combine all neutrons of higher energy into a fast energy group.

TWO GROUP CORE DIFFUSION EQUATIONS

If we consider the fast and thermal energy fluxes, we get as balance equations for the fast and thermal groups:

Fast Group:

-[Leakage]-[Removal by scattering]-[Absorptions]+[Fast fissions]+[Thermal fissions]=0

$$D_1\nabla^2\phi_1 - \Sigma_{1s}\phi_1 - \Sigma_{1a}\phi_1 + \nu\Sigma_{1f}\phi_1 + \nu\Sigma_{2f}\phi_2 = 0 \quad (1)$$

Thermal Group:

-[Leakage]-[Absorptions]+[Downscattering neutrons from fast group]=0

$$D_2\nabla^2\phi_2 - \Sigma_{2a}\phi_2 + \Sigma_{1s}\phi_1 = 0 \quad (2)$$

The equations are coupled through the thermal fission term:

$$\Sigma_{2f}\phi_2,$$

and the fast removal term:

$$\Sigma_{1s}\phi_1.$$

CRITICALITY EQUATION FOR TWO GROUP THEORY AND BARE REACTORS

We assume that the fluxes in the core have a geometrical buckling B^2 satisfying:

$$\nabla^2\phi_1 + B^2\phi_1 = 0$$
$$\nabla^2\phi_2 + B^2\phi_2 = 0$$

Since B^2 is the same for both the thermal and fast fluxes, they are then proportional everywhere for the bare reactor. Equations 1 and 2 can be rewritten as:

$$-D_1 B^2 \phi_1 - \Sigma_{1s}\,\phi_1 - \Sigma_{1a}\,\phi_1 + \nu\,\Sigma_{1f}\,\phi_1 + \nu\,\Sigma_{2f}\,\phi_2 = 0 \qquad (1)'$$

$$-D_2 B^2 \phi_2 - \Sigma_{2a}\,\phi_2 + \Sigma_{1s}\,\phi_1 = 0 \qquad (2)'$$

From Eqn. 2' the ratio of thermal to fast flux is:

$$\frac{\phi_2}{\phi_1} = \frac{\Sigma_{1s}}{D_2 B^2 + \Sigma_{2a}} = \frac{\Sigma_{1s}}{\Sigma_{2a}}\frac{1}{1+L_2^2 B^2}$$
$$where: L_2^2 = \frac{D_2}{\Sigma_{2a}} \qquad (3)$$

Considering the two source terms in Eqn. 1:

$$\nu\,\Sigma_{1f}\,\phi_1 + \nu\,\Sigma_{2f}\,\phi_2 = \frac{\nu\,\Sigma_{1f}\,\phi_1 + \nu\,\Sigma_{2f}\,\phi_2}{\nu\,\Sigma_{2f}\,\phi_2}\nu\,\Sigma_{2f}\,\phi_2 = \varepsilon\nu\,\Sigma_{2f}\,\phi_2 \qquad (4)$$

where ε is the fast fission factor,

$$\varepsilon = 1 + \frac{\Sigma_{1f}\,\phi_1}{\Sigma_{2f}\,\phi_2} \qquad (5)$$

In a bare unreflected reactor core, the ration of thermal to fast flux ratio is a constant given by Eqn. 3, thus:

$$\varepsilon = 1 + \frac{\Sigma_{1f}}{\Sigma_{2f}}\frac{1}{\frac{\Sigma_{1s}}{\Sigma_{2a}}\frac{1}{1+L_2^2 B^2}} = 1 + \frac{\Sigma_{1f}}{\Sigma_{2f}}\frac{\Sigma_{2a}}{\Sigma_{1s}}(1+L_2^2 B^2) \qquad (6)$$

This is not the case in a reflected reactor core since the thermal flux to fast flux ration is not a constant.

Substituting from Eqn. 4 into Eqn. 1' we get

$$+D_1 B^2 \phi_1 + (\Sigma_{1s}\,\phi_1 + \Sigma_{1a})\phi_1 = \varepsilon\nu\,\Sigma_{2f}\,\phi_2$$

Defining the fast diffusion area:

$$L_1^2 = \frac{D_1}{\Sigma_{1s} + \Sigma_{1a}} \qquad (7)$$

we can write:

$$(1+L_1^2B^2) = \frac{\varepsilon\nu\sum_{2f}}{\sum_{1s}+\sum_{1a}}\frac{\phi_2}{\phi_1}$$

$$(1+L_1^2B^2)(1+L_2^2B^2) = \varepsilon\nu\frac{\sum_{1s}}{\sum_{1s}+\sum_{1a}}\frac{\sum_{2f}}{\sum_{2a}} \qquad (8)$$

Since the ratio:

$$\frac{\sum_{1a}}{\sum_{1s}+\sum_{1a}}$$

is the fraction of fast neutrons removed by fast absorptions, we can express the resonance escape probability as:

$$p = 1-\frac{\sum_{1a}}{\sum_{1s}+\sum_{1a}} = \frac{\sum_{1s}}{\sum_{1s}+\sum_{1a}}$$

Since:

$$\eta f = \frac{\nu\sum_{2f}}{\sum_{2a}},$$

$$k_\infty = \eta\varepsilon pf,$$

we can rewrite Eqn. 8 as:

$$k_{eff} = k_\infty\frac{1}{(1+L_1^2B^2)}\frac{1}{(1+L_2^2B^2)} = 1. \qquad (9)$$

For agreement with Fermi Theory, it would be necessary that:

$$e^{-B^2\tau} = \frac{1}{(1+L_2^2B^2)}$$

which can be true only in weakly absorbing media.

CRITICALITY EQUATIONS FOR THE CORE OF A REFLECTED REACTOR

In a reflected system the fast flux to thermal flux ratio is not a constant and the method adopted for a bare unreflected reactor needs to be modified. Equation 1’ can be rewritten in the form:

$$\begin{aligned} &\mathrm{D}_1 B^2 \phi_1 + (\Sigma_{1s} + \Sigma_{1a} - \nu \Sigma_{1f})\phi_1 = \nu \Sigma_{2f} \phi_2 \\ &\mathrm{D}_1 B^2 \phi_1 + \Sigma_{1n} \phi_1 = \nu \Sigma_{2f} \phi_2 \end{aligned} \tag{10}$$

where:

$$\Sigma_{1n} = (\Sigma_{1s} + \Sigma_{1a} - \nu \Sigma_{1f})$$

is a fast net removal cross section from the fast group to the thermal group.

Equation 10 can be rewritten as:

$$\begin{aligned} &(1+\mathrm{L}_n^2 B^2)\frac{\phi_1}{\phi_2} = \frac{\nu \Sigma_{2f}}{\Sigma_{1n}} \\ &where: \mathrm{L}_n^2 = \frac{D_1}{\Sigma_{1n}} \end{aligned} \tag{11}$$

Similarly, Eqn. 3 can be written as:

$$(1+\mathrm{L}_2^2 B^2)\frac{\phi_1}{\phi_2} = \frac{\Sigma_{1s}}{\Sigma_{2a}} \tag{12}$$

Multiplying the sides of Eqns. 11 and 12 we get:

$$(1+L_2^2 B^2)(1+L_n^2 B^2) = \frac{\Sigma_{1s}}{\Sigma_{2a}} \frac{\nu \Sigma_{2f}}{\Sigma_{1n}} = k_n \tag{13}$$

where k_n is a modified multiplication factor. It will be equal to the infinite medium multiplication factor only when there are no fast fissions with:

$$\Sigma_{1f} = 0, \text{ and } \varepsilon = 1 \Rightarrow k_n = k_\infty$$

Equation 13 is a quadratic equation in B^2:

$$\begin{aligned} &(B^2)^2 L_2^2 L_n^2 + (L_2^2 + L_n^2)B^2 - (k_n - 1) = 0 \\ &(B^2)^2 + (\frac{1}{L_n^2} + \frac{1}{L_2^2})B^2 - \frac{(k_n - 1)}{L_2^2 L_n^2} = 0 \end{aligned} \tag{14}$$

If we denote:

$$a = 1, b = (\frac{1}{L_n^2} + \frac{1}{L_2^2}), c = \frac{(k_n - 1)}{L_2^2 L_n^2} = 0$$

then the roots of Eqn. 14 are:

$$+\mu^2 = \frac{-b+\sqrt{b^2+4c}}{2} \tag{15}$$

$$-\nu^2 = \frac{-b-\sqrt{b^2+4c}}{2} \tag{16}$$

By adding Eqns 15 and 16:

$$\nu^2 = \mu^2 + b \tag{17}$$

The root μ^2 is called the principal buckling, and the root ν^2 is called the alternate buckling. Both roots are positive quantities. For a bare reactor only μ^2 is used. If 4c is small compared with b^2, μ^2 will be the difference between two nearly equal quantities, and will not be numerically accurate. In this case one can instead use the binomial expansion:

$$(1+x)^n \approx 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-3)}{3!}x^3 + ...$$

Hence:

$$\begin{aligned}
\mu^2 &= \frac{1}{2}[-b + b\sqrt{1+\frac{4c}{b^2}}] \\
&= \frac{1}{2}[-b + b\left(1+\frac{4c}{b^2}\right)^{\frac{1}{2}}] \\
&= \frac{1}{2}[-b + b + 2\frac{c}{b} - 2\frac{c^2}{b^3} + 4\frac{c^3}{b^5} ...] \\
&= \frac{c}{b}[1 - \frac{c}{b^2} + 2\left(\frac{c}{b^2}\right)^2 + ...]
\end{aligned}$$

THE CORE FLUX DISTRIBUTIONS

Having determined that the buckling B^2 has two values μ^2 and ν^2 as given by Eqns. 15 and 16, we now have two solutions for the flux distributions. For the principal buckling:

$$\nabla^2\phi_{1c} + \mu^2\phi_{1c} = 0$$
$$\nabla^2\phi_{2c} + \nu^2\phi_{2c} = 0$$

Considering spherical geometry, the solutions to these two equations are:

$$\phi_{1c} = AX, X = \frac{\sin \mu r}{r} \tag{18}$$

$$\phi_{2c} = A' X \tag{19}$$

The relationship between A and A’ is found by substituting Eqns. 18 and 19 in Eqn. 2’, yielding:

$$-D_2 \mu^2 A' X - \Sigma_{2a} A' X + \Sigma_{1s} AX = 0$$

Thus we can write for the principal coupling coefficient:

$$S_1 = \frac{A'}{A} = \frac{\Sigma_{1s}}{\mu^2 D_2 + \Sigma_{2a}} \tag{20}$$

Thus Eqns 18 and 19 can be written:

$$\begin{aligned} \phi_{1c} &= AX \\ \phi_{2c} &= AS_1 X \end{aligned} \tag{21}$$

For the solution corresponding to the alternate buckling, similarly:

$$\begin{aligned} \nabla^2 \phi_{1c} - \nu^2 \phi_{1c} &= 0 \\ \nabla^2 \phi_{2c} - \nu^2 \phi_{2c} &= 0 \end{aligned}$$

$$\phi_{1c} = CY, Y = \frac{\sinh \nu r}{r} \tag{22}$$

$$\phi_{2c} = C' Y \tag{23}$$

The alternate coupling coefficient can be obtained by substitution into Eqn. 2’ as;

$$+D_2 \nu^2 C' Y - \Sigma_{2a} C' Y + \Sigma_{1s} CY = 0$$

$$S_2 = \frac{C'}{C} = \frac{\Sigma_{1s}}{\Sigma_{2a} - \nu^2 D_2} \tag{23}$$

If there is a small difference between the terms in the denominator, then we let:

$$
\begin{aligned}
\nu^2 &= \mu^2 + b \\
&= \mu^2 + b \\
&= \mu^2 + \frac{1}{L_n^2} + \frac{1}{L_2^2} \\
&= \mu^2 + \frac{1}{L_n^2} + \frac{\Sigma_{2a}}{D_2}
\end{aligned}
$$

and use:

$$
\begin{aligned}
S_2 &= \frac{\Sigma_{1s}}{\Sigma_{2a} - \nu^2 D_2} \\
&= \frac{\Sigma_{1s}}{\Sigma_{2a} - D_2\mu^2 - \frac{D_2}{L_n^2} - \Sigma_{2a}} \\
&= \frac{\Sigma_{1s}}{D_2(\mu^2 - \frac{1}{L_n^2})}
\end{aligned}
\tag{25}
$$

The alternate solutions then become:

$$
\begin{aligned}
\phi_{1c} &= CY \\
\phi_{2c} &= CS_2Y
\end{aligned}
\tag{26}
$$

From Eqns. 21 and 22 the total solution becomes:

$$\phi_{1c} = AX + CY \tag{27}$$

$$\phi_{2c} = AS_1X + CS_2Y \tag{28}$$

For the two group fluxes in the core a spherical reactor:

$$\phi_{1c} = A\frac{\sin \mu r}{r} + C\frac{\sinh \nu r}{r} \tag{29}$$

$$\phi_{2c} = AS_1\frac{\sin \mu r}{r} + CS_2\frac{\sinh \nu r}{r} \tag{30}$$

Since the flux vanishes at the extrapolated boundary, C=0 for a bare reactor. For a reflected reactor, the flux does not vanish at the extrapolated boundary and C does not equal 0.

The critical size of a bare reactor can be found by using the relationship:

$$\mu^2 = \left(\frac{\pi}{R_{ex}} \right)^2 , \tag{31}$$

but for the reflected reactor, the situation is different.

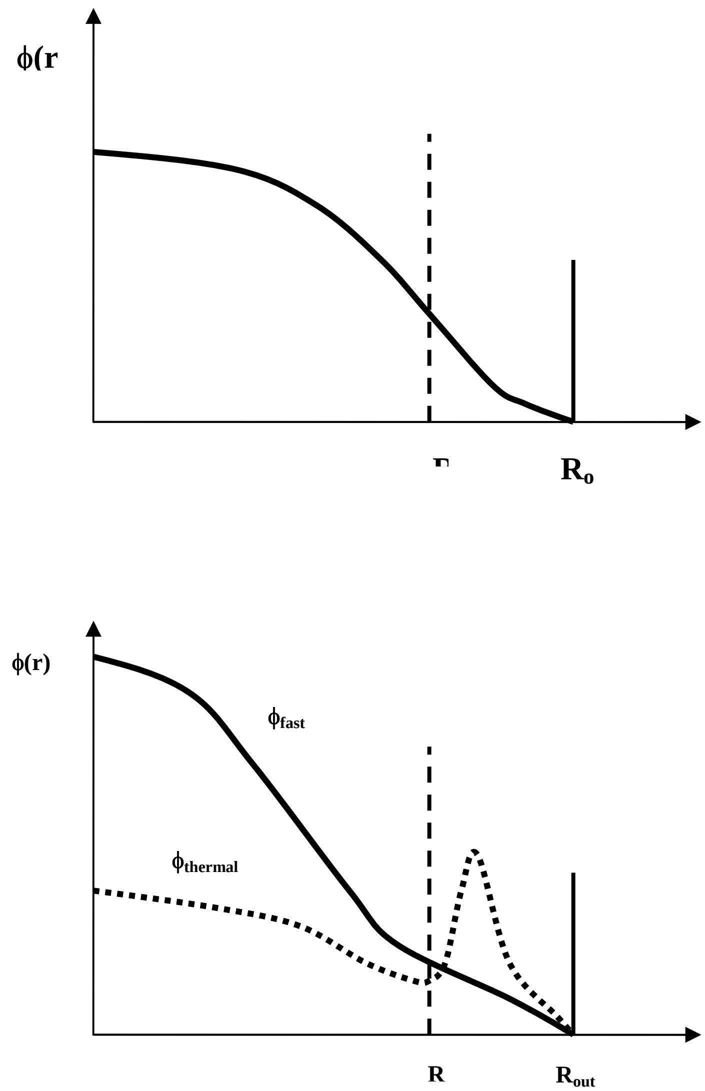

Fig. 1: Flux distributions in the core and reflector of a two region reactor using the one group and the ntwo group models. R_{out}=R+T+d.

SOLUTION IN THE REFLECTOR IN TWO GROUP THEORY

In the reflectror, there is no neutron source and the equations for the fast and thermal groups are:

$$D_{1r}\nabla^2\phi_{1r} - \Sigma_{1sr}\,\phi_{1r} - \Sigma_{1ar}\,\phi_{1r} = 0$$
$$D_{2r}\nabla^2\phi_{2r} - \Sigma_{2ar}\,\phi_{2r} + \Sigma_{1sr}\,\phi_{1r} = 0$$

The source term in the thermal group is here the removal term from the fast group. Dividing by the diffusion coefficients and defining the diffusion areas:

$$L_{2r}^2 = \frac{D_{2r}}{\Sigma_{2ar}}, \qquad L_{1r}^2 = \frac{D_{1r}}{\Sigma_{1sr} + \Sigma_{1ar}} \tag{32}$$

the equations can be written:

$$\nabla^2\phi_{1r} - \frac{1}{L_{1r}^2}\phi_{1r} = 0 \tag{33}$$

$$\nabla^2\phi_{2r} - \frac{1}{L_{2r}^2}\phi_{2r} + \frac{\Sigma_{1sr}}{D_{2r}}\phi_{1r} = 0 \tag{34}$$

Equation 33 has a solution in spherical geometry:

$$\phi_{1r} = FZ_1, Z_1 = \frac{e^{-\frac{r}{L_{1r}}}}{r} \tag{35}$$

for an infinite reflector, but for a finite reflector of thickness T and an extrapolation distance d, it is preferable to use the solution:

$$Z_1 = \frac{\sinh(R + T + d - r)}{r} \tag{36}$$

For the thermal flux solution in the reflector, we write:

$$\phi_{2r} = GZ_2 + S_3\phi_{1r} = GZ_2 + S_3FZ_1 \tag{37}$$

where:

$$Z_2 = \frac{e^{-\frac{r}{L_{2r}}}}{r},$$
$$or:$$
$$Z_2 = \frac{\sinh(R+T+d-r)}{r}$$

Here, S_3 is a coupling coefficient to be determined. It can be noticed that in any geometry:

$$\nabla^2 Z_1 = \frac{1}{L_{1r}^2} Z_1, \nabla^2 Z_2 = \frac{1}{L_{2r}^2} Z_2.$$

Substituting in Eqn. 34, we get:

$$G\frac{Z_2}{L_{2r}^2} + S_3\frac{FZ_1}{L_{1r}^2} - G\frac{Z_2}{L_{2r}^2} - S_3\frac{FZ_1}{L_{1r}^2} + \frac{\sum_{1sr}}{D_{2r}} FZ_1 = 0$$

From which:

$$S_3 = \frac{\frac{\sum_{1sr}}{D_{2r}}}{\frac{1}{L_{2r}^2} - \frac{1}{L_{1r}^2}} \qquad (38)$$

CRITICALITY CONDITION

The continuity of the fluxes and the currents at the interface between the reactor core and reflector can be written as:

$$\begin{aligned} \phi_{1c} &= \phi_{1r} \\ \phi_{2c} &= \phi_{2r} \\ -D_{1c}\nabla\phi_{1c} &= -D_{1r}\nabla\phi_{1r} \\ -D_{2c}\nabla\phi_{2c} &= -D_{2r}\nabla\phi_{2r} \end{aligned} \qquad (39)$$

Substituting in Eqns 39 from Eqns. 27,28, 35 and 37, we get:

$$\begin{aligned} AX + CY &= FZ_1 \\ S_1AX + S_2CY &= S_3FZ_1 + GZ_2 \\ D_{1c}AX' + D_{1c}CY' &= D_{1r}FZ_1' \\ D_{2c}S_1AX' + D_{2c}S_2CY' &= D_{2r}S_3FZ_1' + D_{2r}GZ_2' \end{aligned} \qquad (40)$$

where the primes denote a gradient operation, and the functions are evaluated at r=R.

In matrix notation, these equations can be expressed as:'

$$\begin{pmatrix} X & Y & -Z_1 & 0 \\ S_1 X & S_2 Y & -S_3 Z_1 & -Z_2 \\ D_{1c} X^{'} & D_{1c} Y^{'} & -D_{1r} Z_1^{'} & 0 \\ D_{2c} S_1 X^{'} & D_{2c} S_2 Y^{'} & -D_{2r} S_3 Z_1^{'} & -D_{2r} Z_2^{'} \end{pmatrix} \begin{pmatrix} A \\ C \\ F \\ G \end{pmatrix} = 0 \qquad (41)$$

These are four simultaneous linear algebraic equations in the four unknowns A, C, F and G. These equations are of the homogeneous type being equal to zero on the right hand side, and it is not possible to obtain explicit expressions for all four constants. As in all sterady state reactor problems, there will remain one undetermined constant corresponding to the flux or the power level in the reactor. Thus three of these unknowns can be determined in terms of the fourth.

The only non trivial solution of this linear system of equations requires that the determinant of the coefficients vanishes:

$$\Delta \equiv \begin{vmatrix} X & Y & -Z_1 & 0 \\ S_1 X & S_2 Y & -S_3 Z_1 & -Z_2 \\ D_{1c} X^{'} & D_{1c} Y^{'} & -D_{1r} Z_1^{'} & 0 \\ D_{2c} S_1 X^{'} & D_{2c} S_2 Y^{'} & -D_{2r} S_3 Z_1^{'} & -D_{2r} Z_2^{'} \end{vmatrix} = 0 \qquad (42)$$

This expression may be regarded as the criticality condition for the reflected reactor in the two group formulation, since it involves both the group constants and the dimensions of the reactor. The specifications of the fuel concentration or of the core size allows the determination of the other through the use of the above relation.

EVALUATION OF CRITICALITY DETERMINANT

The evaluation of the criticality relation is laborious, although straightforward. Some simplification in the calculation can be achieved if one takes advan tage of the fact that many terms in the determinant are insensitive to variations in the fuel concentration or the size of the core.

Dividing the first column by X, the second by Y, the third by Z_1, and the fourth by Z_2, then the third row by D_{1c} and the fourth row by D_{2c}, does not change the value of the determinant and we get:

$$\Delta \equiv \begin{vmatrix} 1 & 1 & -1 & 0 \\ S_1 & S_2 & -S_3 & -1 \\ \alpha & \beta & -\rho_1\gamma & 0 \\ S_1\alpha & S_2\beta & -\rho_2 S_3\gamma & -\rho_2\delta \end{vmatrix} = 0$$

$$where: \alpha = \frac{X'}{X}, \beta = \frac{Y'}{Y}, \gamma = \frac{Z_1'}{Z_1}, \delta = \frac{Z_2'}{Z_2}$$

$$\rho_1 = \frac{D_{1r}}{D_{1r}}, \rho_2 = \frac{D_{2r}}{D_{2r}}$$

Proceeding to solve in terms of the minors of the fourth column:

$$\Delta = -1\begin{vmatrix} 1 & 1 & -1 \\ \alpha & \beta & -\rho_1\gamma \\ S_1\alpha & S_2\beta & -\rho_2 S_3\gamma \end{vmatrix} - \rho_2\delta \begin{vmatrix} 1 & 1 & -1 \\ S_1 & S_1 & -S_3 \\ \alpha & \beta & -\rho_1\gamma \end{vmatrix} = 0$$

Using the scissors rule for expanding the determinants we get:

$$\begin{aligned} \alpha &= \frac{\rho_2\delta C_1 + \rho_1\gamma C_2 + \beta C_3}{C_1 + C_2 + C_3} \\ where:\ C_1 &= S_1(\rho_1\gamma - \beta) \\ C_2 &= S_2(\beta - \rho_2\delta) \\ C_3 &= S_3\rho_2(\delta - \gamma) \end{aligned} \tag{43}$$

This is the two group critical equation and is satisfied by only the critical dimension. The computational procedure to be followed for a given reactor size in using Eqn. 43 consists in using various values of the fuel concentration, and evaluating each side of Eqn. 3. The RHS and the LHS can then be plotted as a function of the fuel concentration, and the intersection of the two curves yields the critical concentration.

Another type of calculation would be to determine the critical reactor size for a given reactor composition. The functions X, Y, Z_1 and Z_2 and their gradients in spherical geometry are listed in Table 1.

Table 1: Solution functions for reflected cores in spherical geometry. R is core radius, T is reflector thickness and d is extrapolation length.

X(r)	$\frac{\sin \mu r}{r}$
X'(r)	$\frac{\mu \cos \mu r}{r} - \frac{\sin \mu r}{r^2}$

Y(r)	$\frac{\sinh \nu r}{r}$
Y'(r)	$\frac{\nu \cosh \nu r}{r} - \frac{\sinh \nu r}{r^2}$
Z_1(r)	$\frac{\sinh \frac{R+T+d-r}{L_{1r}}}{r}$
Z_1'(r)	$\frac{-\frac{1}{L_{1r}}\cosh \frac{R+T+d-r}{L_{1r}}}{r} - \frac{\sinh \frac{R+T+d-r}{L_{1r}}}{r^2}$
Z_2(r)	$\frac{\sinh \frac{R+T+d-r}{L_{2r}}}{r}$
Z_1'(r)	$\frac{-\frac{1}{L_{2r}}\cosh \frac{R+T+d-r}{L_{2r}}}{r} - \frac{\sinh \frac{R+T+d-r}{L_{2r}}}{r^2}$

DETERMINATION OF THE FLUX DISTRIBUTIONS

If the reactor critical composition or size are determined, the constants C, F, and g can now be evaluated in terms of A. We first rewrite Eqn. 40 in the form:

$$AX + CY - FZ_1 = 0 \quad (44)$$

$$S_1AX + S_2CY - S_3FZ_1 - GZ_2 = 0 \quad (45)$$

$$AX' + CY' - \rho_1 FZ_1' = 0 \quad (46)$$

$$S_1AX' + S_2CY' - \rho_2 S_3 FZ_1' - \rho_2 GZ_2' = 0 \quad (47)$$

Dividing Eqn. 46 by Eqn. 44:

$$\frac{AX' + CY'}{AX + CY} = \frac{\rho_1 FZ_1'}{FZ_1} = \rho_1 \gamma$$

From which we can estimate C:

$$C = \frac{\rho_1\gamma - \alpha}{\beta - \rho_1\gamma}\frac{X}{Y}A \quad (48)$$

Substituting from Eqns. 44 and 45 into Eqn. 45 gives:

$$F = \frac{\beta - \alpha}{\beta - \rho_1 \gamma} \frac{X}{Z_1} A \quad (49)$$

Substituting from Eqns. 44 and 45 into Eqn. 45 gives:

$$G = \frac{S_1(\beta - \rho_1\gamma) + S_2(\rho_1\gamma - \alpha) - S_3(\beta - \alpha)}{\beta - \rho_1\gamma} \frac{X}{Z_2} A \quad (50)$$

Equation 47 yields no new information, and we now have C, F and G in terms of A. The constant A can be determined as a function of the reactor power level in the fo;;owing manner. The reactor power can be written as:

$$\begin{aligned} P[\text{Watts}] &= (\Sigma_{1f}\,\bar{\phi}_{1c} + \Sigma_{2f}\,\bar{\phi}_{2c}) \frac{\text{fissions}}{\text{cm}^3.\text{sec}} .V \ \text{cm}^3 .200 \frac{\text{MeV}}{\text{fission}} 1.6x10^{-13} \frac{\text{Joule}}{\text{MeV}} \\ &= 3.2x10^{-11}(\Sigma_{1f}\,\bar{\phi}_{1c} + \Sigma_{2f}\,\bar{\phi}_{2c})V \\ &= 3.2x10^{-11}\varepsilon\,\Sigma_{2f}\,\bar{\phi}_{2c}V \end{aligned} \quad (51)$$

where:

$$\begin{aligned} \bar{\phi}_{1c} &= \frac{\int_V \phi_{1c} dV}{\int_V dV} = \frac{A\int_V XdV + C\int_V YdV}{\int_V dV} \\ \bar{\phi}_{2c} &= \frac{\int_V \phi_{2c} dV}{\int_V dV} = \frac{AS_1\int_V XdV + CS_2\int_V YdV}{\int_V dV} \end{aligned} \quad (52)$$

ε is the fast fission factor.

In spherical geometry the element of volume dV is:

$$dV = 4\pi r^2 dr$$

from which:

$$\begin{aligned} \bar{\phi}_{1c} &= \frac{4\pi A\int_0^R r\sin\mu r dr + 4\pi C\int_0^R r\sinh\nu r dr}{\frac{4}{3}\pi R^3} \\ &= \frac{3A}{\mu^2 R^3}(\sin\mu R - \mu R\cos\mu R) + \frac{3C}{\nu^2 R^3}(-\sinh\nu R + \nu R\cosh\nu R) \end{aligned}$$

This can be rewritten as:

$$\bar{\phi}_{1c} = \frac{3}{R}\frac{A}{\mu^2}\frac{\sin\mu R}{R}(\frac{1}{R} - \mu\cot\mu R) + \frac{3}{R}\frac{C}{\nu^2}\frac{\sinh\nu R}{R}(-\frac{1}{R} + \nu\coth\nu R)\,.$$

But since:

$$\alpha = \frac{X^{'}}{X} = \mu\cot\mu R - \frac{1}{R}$$
$$\beta = \frac{Y^{'}}{Y} = \nu\coth\nu R - \frac{1}{R}$$

thus, using the expression 48 for C, we get:

$$\begin{aligned}
\bar{\phi}_{1c} &= -\frac{3}{R}\frac{A}{\mu^2}\frac{\sin\mu R}{R}\alpha + \frac{3}{R}\frac{1}{\nu^2}\frac{\rho_1\gamma - \alpha}{\beta - \rho_1\gamma}\frac{X}{Y}A\frac{\sinh\nu R}{R}\beta \\
&= -\frac{3}{R}\frac{A}{\mu^2}X\alpha + \frac{3}{R}\frac{1}{\nu^2}\frac{\rho_1\gamma - \alpha}{\beta - \rho_1\gamma}\frac{X}{Y}AY\beta \\
&= \left[-\frac{\alpha}{\mu^2}(\beta - \rho_1\gamma) + \frac{\beta}{\nu^2}(\rho_1\gamma - \alpha)\right]\frac{3X}{R(\beta - \rho_1\gamma)}A
\end{aligned}$$

The average value of the thermal flux in the core is found similarly to be:

$$\bar{\phi}_{2c} = \left[-\frac{S_1\alpha}{\mu^2}(\beta - \rho_1\gamma) + \frac{S_2\beta}{\nu^2}(\rho_1\gamma - \alpha)\right]\frac{3X}{R(\beta - \rho_1\gamma)}A$$

From Eqn. 51, we now get:

$$P[\text{Watts}] = 3.2x10^{-11}\varepsilon\Sigma_{2f}\left[-\frac{S_1\alpha}{\mu^2}(\beta - \rho_1\gamma) + \frac{S_2\beta}{\nu^2}(\rho_1\gamma - \alpha)\right]\frac{3X}{R(\beta - \rho_1\gamma)}A.\frac{4\pi R^3}{3}$$

From which:

$$A = \frac{P}{3.2x10^{-11}\varepsilon\sum_{2f}.4\pi R^2}\frac{(\beta-\rho_1\gamma)}{X}\frac{1}{\left[-\frac{S_1\alpha}{\mu^2}(\beta-\rho_1\gamma)+\frac{S_2\beta}{\nu^2}(\rho_1\gamma-\alpha)\right]}$$

$$= \frac{P}{3.2x10^{-11}\varepsilon\sum_{2f}.4\pi R^2}\frac{(\beta-\rho_1\gamma)}{X}\frac{1}{(S_1m_1+S_2m_2)}$$

$$where: m_1 = -\frac{\alpha}{\mu^2}(\beta-\rho_1\gamma)$$

$$m_2 = \frac{\beta}{\nu^2}(\rho_1\gamma-\alpha)$$

Knowing A in terms of the reactor power, and the constants C, F, anf G in terms of A, one can n ow plot the flux distributions in the core and the reflector. These would be needed for the heat transfer calculations since the neutron flux distribution determines the heat generation distribution. They would also be needed for the fuel burnup and management as well as the radiation damage and materials considerations.

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