THE RESONANCE ESCAPE PROBABILITY

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INTRODUCTION

We use the Breit-Wigner single level resonance formula for the absorption cross section as a function of energy, and the expressions for the slowing down density to derive an equation for the probability for a neutron to escape being absorbed in the resonance region as it slows down to thermal energies. The resulting expression for the resonance escape probability forms one term in the four factor formula for the infinite medium multiplication factor. Expressions for the resonance integral for homogeneous reactors are also derived.

THE BREIT-WIGNER SINGLE LEVEL RESONANCE FORMULA

For a single level resonance, the Breit-Wigner formula can be written as:

$$\sigma_a(E) = \sigma_0 \left(\frac{E_0}{E}\right)\frac{1}{1+\left[\frac{(E-E_0)}{\Gamma/2}\right]^2} \quad (1)$$

where: σ_0 and E_0 are the cross section and energy at the peak of the resonance, and Γ is the width of the resonance at half maximum, as shown in Fig. 1.

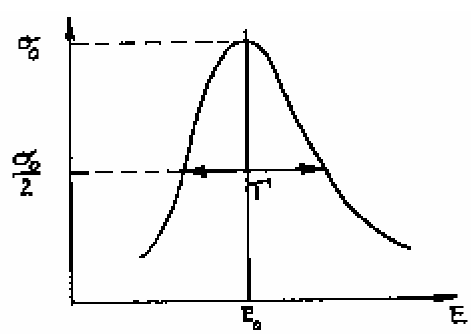

Fig. 1 Single-level resonance.

For a broad resonance, $\Gamma > (E - E_0)$,

$$\sigma_c = \sigma_0 \left(\frac{E_0}{E} \right)^{1/2} = \frac{const}{v},$$

which is the known 1/v dependence, e.g. Boron.

RESONANCE ESCAPE PROBABILITY

The probability of escaping absorption in the resonances while slowing down is called the resonance escape probability and is by definition:

$$P(E) = \frac{q(E)}{q_0} \tag{2}$$

where q_0 is the slowing-down density at the fission energy.

In an infinite medium the change in slowing down density in an energy interval dE is equal to the number of absorptions in dE, or:

$$\frac{dq}{dE}.dE = \Sigma_a \phi dE \tag{3}$$

We have proven before that the number of collisions per unit volume per unit time in the absence of absorptions was:

$$\frac{q}{\xi E} dE = \Sigma_s \phi dE$$

If we now include the absorptions, we can write:

$$\frac{q}{\xi E} dE = (\Sigma_s + \Sigma_a) \phi dE \tag{4}$$

Dividing Eqns. 3 and 4, we get:

$$\frac{\Sigma_a}{(\Sigma_s + \Sigma_a)} = \frac{\frac{dq}{dE}}{q / \xi E}$$

or:

$$\frac{dq}{q} = d[\ln q(E)] = \frac{\Sigma_a}{\xi(\Sigma_s + \Sigma_a)} \frac{dE}{E}.$$

If we integrate from the fission energy E_0 to an arbitrary energy E, we get:

$$\ln \frac{q(E)}{q(E_0)} = \int_{E_0}^{E} \frac{\Sigma_a}{\xi(\Sigma_s + \Sigma_a)} \frac{dE}{E}$$

From Eqn. 2, and noting that $q(E_0)=q_0$, thus:

$$p(E) = \frac{q(E)}{q_0} = e^{-\int_{E}^{E_0} \frac{\Sigma_a}{\xi(\Sigma_s + \Sigma_a)} \frac{dE}{E}} \qquad (5)$$

At thermal energy:

$$p = \exp(-\int_{E_{th}}^{E_0} \frac{\Sigma_a}{\xi(\Sigma_s + \Sigma_a)} \frac{dE}{E}) \qquad (6)$$

THE RESONANCE INTEGRAL FOR HOMOGENEOUS REACTORS

To estimate p from Eqn. 6, one needs to know the energy dependence of the cross sections and integrate over the whole energy range. For homogeneous reactors with weak fast absorptions we can approximate the integral in Eq. 6. Since Σ_s is fairly constant we can write it as:

$$\frac{1}{\xi\overline{\Sigma}_s}\int_{E_{th}}^{E_0}\frac{\Sigma_a}{1+\frac{\Sigma_a}{\Sigma_s}}\frac{dE}{E}$$

If we have a very predominant absorber (e.g U^{235}), we can substitute $N\sigma_a$ for Σ_a and rewrite the integral as:

$$\frac{N_a}{\xi\overline{\Sigma}_s}\int_{E_{th}}^{E_0}\frac{\sigma_a}{1+\frac{N_a\sigma_a}{\Sigma_s}}\frac{dE}{E} \qquad (7)$$

In case of "infinite dilution" of the absorbing material:

$$\Sigma_s >> \Sigma_a,$$

and we can write:

$$p = \exp(-\frac{N_a I_0}{\xi\overline{\Sigma}_s}) \qquad (8)$$

wher:e $I_0 = \int_{E_{th}}^{E_0}\sigma_a\frac{dE}{E}$ is the infinite dilution resonance integral.

If Σ_s is comparable to Σ_a, we have the effective resonance integral:

$$I_{eff} = \int_{E_{th}}^{E_0}\frac{\sigma_a}{1+\frac{N_a\sigma_a}{\sigma_s}}\frac{dE}{E} \qquad (9)$$

If we remove the $(1+\frac{N_a\sigma_a}{\Sigma_s})$ term from the denominator:

$$I_{eff} = \frac{\int_{E_{th}}^{E_0} \sigma_a \frac{dE}{E}}{1 + \frac{N_a \sigma_a}{\overline{\Sigma_s}}} = \frac{I_0}{1 + \frac{N_a \overline{\sigma_a}}{\overline{\Sigma_s}}}$$

where: $\overline{\sigma_a}$ and $\overline{\Sigma_s}$ are average values over the energy range.

In this case:

$$\overline{\sigma_a} = \frac{\int_{E_{th}}^{E_0} \sigma_a \phi(E) dE}{\int_{E_{th}}^{E_0} \phi(E) dE}$$

If $\phi(E) = \phi_0/E$, we get:

$$\overline{\sigma_a} = \frac{\int_{E_{th}}^{E_0} \sigma_a \frac{dE}{E}}{\int_{E_{th}}^{E_0} \frac{dE}{E}} = \frac{I_0}{\ln\left(\frac{E_0}{E_{th}}\right)}$$

Thus we can write for the effective resonance integral:

$$I_{eff} = \frac{I_0}{1 + \frac{N_a}{\Sigma_s} \frac{I_0}{\ln\left(\frac{E_0}{E_{th}}\right)}} \qquad (10)$$

and the resonance escape probability is now written as:

$$p = \exp(-\frac{N_a I_{eff}}{\xi \overline{\Sigma_s}}) \qquad (11)$$

This equation is valid for homogeneous reactors. Some values of I_0 are in Table 1.

Table 1: Values for infinitely dilute resonance integrals.

Nuclide	**Process**	**I_0** **(0.5-1.0 MeV)**	**I'_0** **(0.5-1.0 MeV)**	$\ln\left(\frac{E_0}{E_{th}}\right)$
$_{90}Th^{232}$	capture	85	-	14.5
$_{90}Th^{233}$	capture	400	-	14.5
$_{91}Pa^{233}$	capture	1200	-	14.5
$_{92}U^{233}$	capture	300	-	14.5
	absorption	1200	-	14.5
	fission	900	-	14.5
$_{92}U^{234}$	capture	700	-	14.5
$_{92}U^{235}$	capture	150	-	14.5
	absorption	450	-	14.5
	fission	300	-	14.5
$_{92}U^{236}$	capture	400	-	14.5
$_{92}U^{239}$	capture	280	-	14.5
$_{94}Pu^{239}$	capture	-	1500	15.7
	absorption	-	3500	15.7
	fission	-	2000	15.7
$_{94}Pu^{240}$	capture	9000	-	14.5
$_{94}Pu^{241}$	capture	-	1000	15.7
	absorption	-	2800	15.7
	fission	-	1800	15.7
$_{94}Pu^{242}$	capture	1300	-	14.5

absorption = fission + capture

REFERENCES

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2. J. R. Lamarsh, “Introduction to Nuclear Engineering,” Addison-Wesley Publishing Company, 1983.